

# A STUDY OF A PROPERTY OF PISOT NUMBERS

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# Abstract

In this thesis I studied an approximation property of Pisot numbers. The thesis consists of three chapters.

The first chapter is devoted to recall some properties of the set of Pisot numbers and the set of beta-numbers. I also recalled the representation of a real number in a real basis and I showed some properties of the beta-numbers.

In the second chapter, I gave a complete proof of the theorem related to the approximation property of the Pisot numbers proved by Bugeaud. I showed some results from automata theory and I derived a complete proof of the approximation property theorem.

In the last chapter, I obtain some optimal values of the quantities  $l_m(q)$  when  $q$  runs through certain intervals. Recall that  $l_m(q)$  is the infimum of the positive numbers  $|P(q)|$ , where  $P$  is a non-zero polynomial with rational integer coefficients of absolute value at most  $m$  and  $m$  is a natural number. I used the algorithm of Borwein and Hare to compute the numbers  $l_m(q)$  for some Pisot numbers  $q$  and some values of  $m$ . This computation allowed me to determine the maximum of the numbers  $l_m(q)$  when  $2 < q < 3$  and  $m < 12$ . I also determined the maximum of the quantities  $l_{m+1}(q)$  when  $m < q < m+1$  for all  $m > 1$ . The thesis contains also some secondary results which are not mentioned here.

The thesis ends with a list of references.