

King Saud University  
College of Engineering  
Department of Industrial Engineering

# **Modeling Optimal Water Allocation Under Deficit Irrigation**

BY  
*Engr. Khaled Al-Sitt*

Supervised by:  
*Dr. Mohamad N. Azaiez*

Submitted in Partial Fulfillment of the Requirements  
for the Degree of Master of Science in Industrial  
Engineering in the College of Engineering, Department  
of Industrial Engineering, King Saud University

Riyadh  
Rabi'I 1424 H.  
April 2004

# **In the Name of ALLAH**

**We hereby approved the thesis entitled:**

## **Modeling Optimal Water Allocation Under Deficit Irrigation**

**BY**  
*Engr. Khaled Al-Sitt*

### **Committee Members:**

**Supervisor:**

\_\_\_\_\_  
**Dr. Mohamed N. Azaiez**

**Examiner:**

\_\_\_\_\_  
**Dr. Moncer Hariga**

**Examiner:**

\_\_\_\_\_  
**Dr. Ibrahim Al-Harkan**

# Abstract

---

Deficit irrigation consists on deliberately applying an amount of water at a level below water requirement during the irrigation season in order to increase the area irrigated using the same amount of water. The principal reasons for considering deficit irrigation are limited supplies of water and high water costs. Therefore, for regions that suffer from water scarcity such as the Kingdom of Saudi Arabia (KSA), it is beneficial to apply deficit irrigation for various crops in order to maximize the yield/benefit by extending the irrigated area while using the available irrigation water.

In this study, we develop four models of optimal water allocation with deficit irrigation in order to determine the optimal cropping plan for a variety of scenarios. The first model (Dynamic programming model (DP)) allocates a given amount of water optimally over the different growth stages to maximize the yield per hectare for a given crop, accounting for the sensitivity of the crop growth stages to water stress.

The second model (Single Crop Model) tries to find the best allocation of the available water both in time and space in order to maximize the total expected yield of a given crop. It uses several sub-models that interface with each other and with the DP to generate the optimal trade off between irrigation levels and irrigated area. Moreover, it provides intermediate output used by the third model.

The third model (Multi-crop Model) is an optimization model that determines the optimal allocation of land and water for different crops. It shows the importance of several factors in producing an optimal cropping plan. These include water availability, crop predecessor coefficient, market and land constraints and profit values of competing crops.

The output of the models is prepared in a readable form to the normal user by the fourth model (Irrigation Schedule Model).

All the proposed models were coded using visual basic 6.0 programming language

## **Acknowledgement**

Above all, I thank Allah Almighty for his mercy.

I would like to take a moment to express my deep appreciation and gratitude to my family and particularly to my parents for their invaluable support. I also would like to extend my thanks to my fiancé for her support.

Moreover, I would like to express my gratefulness and acknowledgment to my thesis advisor Dr. Naceur Azaiez for his guidance and patience, to Dr Ibrahim Al-Harkan, for his generous support through all the my master career, to Dr. Moncer Hariga for his constructive comments, and to Dr. Abdulrahman Alazba for his appreciated support, help and time.

Khaled Al-Sitt

## ملخص الرسالة

---

يعتمد الري الناقص على استخدام كمية من الماء أقل من الاحتياجات المائية خلال فترة الري بهدف زيادة المساحة المروية عند وجود نقص في مياه الري. تقليل مياه الري للمحاصيل يقلل المحصول الناتج من وحدة المساحة المزروعة و لكن يسمح بزيادة المساحات المروية و ربما زيادة المحصول الكلي.

الأسباب الرئيسية التي تدفع لاستخدام الري الناقص هي قلة مياه الري و ارتفاع أسعارها, لهذا فإن تطبيق مبادئ الري الناقص في المناطق التي تعاني من ندرة المياه مثل المملكة العربية السعودية يمكن أن يكون مفيدا في زيادة الإنتاج الربح عبر استخدام مياه الري المتوفرة بطريقة مجدية. في هذه الدراسة قمنا بإنشاء أربعة نماذج رياضية لتوزيع المياه في حالة الري الناقص من أجل تحديد خطة الزراعة المثلى. النماذج المقترحة تساعد المزارعين و الشركات الزراعية على الإجابة على عدد من الأسئلة الصعبة التي تظهر خلال عملية التخطيط في بداية كل موسم زراعي, هذه الأسئلة هي: ما هي المحاصيل التي يجب زراعتها؟ ما هي مساحة الأرض المخصصة لكل محصول؟ ما هي مستويات الري التي يجب تحقيقها لكل محصول خلال فترة نموه و بالتالي ما هي كمية المياه التي سيستهلكها كل محصول؟ ما هي كمية الإنتاج المتوقعة لكل محصول؟ و في النهاية ما هو الربح الكلي المتوقع في نهاية الموسم الزراعي؟ النماذج المقترحة تأخذ بعين الاعتبار عدد من العوامل التي تؤثر على اختيار خطة الزراعة مثل: مساحة الأراضي المتوفرة عبر مختلف المواسم,

التوزيع السابق للأراضي المتوفرة, كمية مياه الري المتوفرة, احتياجات السوق من المحاصيل.

النموذجين الأول و الثاني مخصصان للتعامل مع محاصيل منفردة مع الأخذ بعين الاعتبار الأراضي التي ستزرع و هي إما محدودة المساحة أو ذات مساحة غير محدودة بينما يتعامل النموذج الثالث مع عدة محاصيل تتنافس من أجل الأراضي و مياه الري في نفس الوقت.

النموذج الأول يستخدم البرمجة الديناميكية و يقوم بإيجاد التوزيع الأمثل لكمية محددة من مياه الري على مختلف مراحل النمو لمحصول محدد من اجل الحصول على أكبر قدر ممكن من الإنتاج لوحدة من المساحة مع الأخذ بعين الاعتبار حساسية مختلف مراحل النمو تجاه نقص المياه. النموذج الثاني (نموذج المحصول الواحد) يبحث عن أفضل توزيع لمياه الري خلال الوقت و المكان من أجل إيجاد أكبر قدر ممكن من الانتاج الكلي. هذا النموذج يستخدم النموذج الأول من اجل تحديد توزيع المياه على مراحل النمو المختلفة للمحصول الذي تتم دراسته. نتائج هذا النموذج تظهر أن الري الناقص يمكن أن يكون مفيداً لبعض المحاصيل و ضار للبعض الآخر, كذلك يظهر العلاقة بين زيادة الأراضي المزروعة و بين كمية المياه المخصصة لوحدة المساحة من الأراضي المزروعة مثل: كمية المياه المتوفرة, عامل التتابع بين المحاصيل, متطلبات السوق من كل محصول, الربح الناتج عن كل محصول.

كما تم تطوير نموذج رابع (نموذج خطة الري) يمكن من عرض نتائج الخطة المثلى للري بطريقة بسيطة و مقروءة.

# Table of Contents

---

List of Tables .....	I
List of Figures .....	III
Introduction.....	9
Literature Review .....	13
Problem Description .....	18
3.1 Scope.....	18
3.2 Problem Definition .....	18
The Dynamic Programming Model.....	22
4.1. Overview of the model .....	22
4.2. Objective of the model.....	23
4.3. Inputs .....	23
4.3. Outputs.....	23
4.5. Notations.....	24
4.5.1. Model Parameters:.....	24
4.6. The elements of the dynamic program .....	24
4.7. The multiplicative form .....	25
4.8. The additive form.....	28
4.9. Illustrations .....	28
Example 4.1 .....	29
Example 4.2 .....	30
The Single Crop Model.....	33
5.1. Overview of the model .....	33
5.2. Objective of the Model: .....	34
5.3. Methodology:.....	34
5.4. Season Sub-model.....	35
5.4.1. Overview.....	35
5.4.2. Objective.....	36
5.4.3. Inputs .....	36
5.4.4. Outputs.....	36
5.4.5. Notations .....	36
5.4.6. The procedure .....	37
Example 5.1 .....	40
5.5 Water-Yield Sub-Model .....	41
5.5.1. Overview.....	41
5.5.2. Objective.....	41
5.5.3. Inputs .....	41
5.5.4. Outputs.....	42
Example 5.2 .....	42
5.6 Control Sub-Model .....	44

5.6.1. Overview.....	44
5.6.2. Objective.....	44
5.6.3. Inputs.....	44
5.6.4. Outputs.....	45
5.6.5. Notations.....	45
5.6.6. Methodology.....	46
5.6.7. Formulation.....	46
Example 5.3.....	53
5.7 Interfacing with the Multi-crop Model.....	65
The Multi-Crop Model.....	66
6.1. Overview of the model.....	66
6.2. Objective of the model.....	67
6.3. Methodology.....	67
6.4. Inputs.....	68
6.5. Outputs.....	69
6.5.1. General output.....	69
6.5.2. Detailed output.....	69
6.6. Notations.....	70
6.6.1. Land notations.....	70
6.6.2. Irrigation Season Notations.....	70
6.6.2. Crop Notations.....	71
6.6.3. Decision variables.....	72
6.7. Objective Function.....	73
6.8. Constraints.....	74
6.8.1. Market constraints.....	74
6.8.2. Land constraints.....	75
6.8.3. Water Constraints.....	75
6.8.4. Time constraints.....	76
6.8.5. Mass balance constraints.....	79
6.9. Illustration.....	81
6.9.1 Discussion of the results.....	83
6.9.2 Sensitivity Analysis.....	84
Irrigation Schedule Model.....	87
7.1. Overview of the model.....	87
7.2. The input for this model.....	87
7.3. The output of this model.....	87
7.4. Notations.....	88
7.5. Formulation.....	88
Example 7.1.....	89
Conclusions and Directions for Future Research.....	91
8.1. Conclusions.....	91
8.2. Direction for Future Research.....	92
Appendices.....	96

# Chapter 1

---

## Introduction

Various factors influence the level of profit attainable from irrigated crops. Some of these are beyond the control of the farmer, such as rainfall. Others are determined by various interactions related to how, when, and how much irrigation water is applied. Irrigation requires some relatively high investment in equipment, fuel, maintenance and labor, but offers a significant potential for increasing net farm income. Irrigation scheduling consists on applying the right amount of water at the right time, usually to meet crop water requirements. These depend basically on the crop life stage. In general, the following five crop growth stages were widely stated in the literature [14], [6]

- 1- Establishment
- 2- Vegetative
- 3- Flowering
- 4- Yield formation
- 5- Ripening.

Some crops however may have smaller number of growth stage ranging between 3 and 5 stages [4]. Water requirement and duration of each stage differ as the growth rate differs from one stage to the other. This is true both for the same crop and for different crops. The amount of water applied to the crops during the irrigation periods has a major impact both on the growth of a crop and on its yield. Each crop has optimal amount of water that results in maximum growth and maximum yield. When water supply is sufficient, it is important to apply the right amount of water to satisfy

crop requirement. In fact, both excessive and deficit irrigation will have a negative effect on the growth and consequently on the yield.

Applying excessive irrigation water will increase the depth of infiltrated water until no more water can be absorbed. This will prevent root aeration and necessitates installing a drainage system to get rid of the unabsorbed water. The drainage water will cause nutrient loss, which will affect negatively the crop yield. Moreover, the cost of the drainage system (operating, maintenance, labor, etc) will be added to the overall cost of the crops reducing the corresponding profit.

When water supply does not meet water requirements during some of the crop growth stages, the actual evapotranspiration will fall below maximum evapotranspiration. Under this condition, water stress will develop in the plants, which will adversely effect crop growth and ultimately crop yield.

The effect of water stress on growth and yield depends on the magnitude and time of occurrence of water stress. It also differs with crop species and variety. When water deficit occurs during a particular part of the total growing period of the crop, the yield response to water deficit can vary greatly depending on how sensitive the crop is to water stress at the particular growth period. In general, crops are more sensitive to water deficit during emergence, flowering and early yield formation than they are during early vegetative, after establishment and late growth periods (ripening). Therefore, the planning of seasonal supply for the available amount of water must be directed towards meeting the full water requirements of the crop during the most sensitive growth stages rather than spreading the available limited supply to the crop equally over the total growing periods. For the non sensitive growth stages, the amount of water allocated must be sufficient enough to prevent productive tissue to die at any stage. Different crops resist differently to water shortages. Some may have a major yield reduction when it is subject to water deficit, while others may have a minor yield reduction.

The effect of water deficit on crop growth and crop yield can be quantified by empirically derived values of what is called yield response factor,  $k_y$  [6]. Large values of this factor indicate that the growth and yield will be greatly affected by water deficit and vice-versa. Under the same conditions of limited water that is spread equally over

the total growing season, involving crops with different  $ky$  values, the crop with the higher  $ky$  value will suffer a greater yield loss than the crop with the lower  $ky$  values. Therefore, when we want to maximize the production in some area that has different crops, the available water supply should be directed towards meeting the full water requirement of the crops that have the highest  $ky$  values in a given period of time. However, for the crops with low  $ky$  values increasing the yield may be achieved by increasing the total cropped area without meeting necessarily the full water requirements.

When the objective is to increase the net farm income, which is a legitimate objective in all projects, the irrigation should be directed towards increasing the production of the most profitable crops. Therefore, when having two different crops that require the same amount of water and have the same response to water deficit (the same yield response factors) the priority of water allocation in case of shortage should be given to the crop with the larger economic value.

Irrigation planning is commonly predicted on complete avoidance of water deficit. Thus, years of short water supply suggest that irrigated acreage be reduced so that water requirement of each acre can be met in full in order to have the maximum yield. This would be inadequate as water shortages become more common and irrigation prices rise. Satisfying full water requirement maximizes the yield and therefore the profit per unit area cropped, but this need not lead to the optimal profit. Deficit irrigation may reduce the yield per unit area cropped but enables to increase the irrigated area. Therefore, the loss in yield per unit area can be overstepped by yield coming out from the additional irrigated area. Therefore, the total profit may increase due to increasing the total yield while using the same amount of water.

One should differentiate between applying water deficit irrigation principles according to unexpected water shortages in order to minimize the total loss resulting from the water shortage and deliberately under-irrigating in order to increase the irrigated area and possibly the profit using the same amount of water. Deficit irrigation principles appear to be very important in arid areas such as Saudi Arabia, since it enables to minimize the effects of water scarcity and may maximize the yield and therefore the profit for some restricted amount of water.

In this study, we develop four models of optimal water allocation with deficit irrigation in order to determine the optimal cropping plan for a variety of scenarios. The models help the farmers and the agricultural companies answer a number of difficult questions that arise at the beginning of each cropping season. The questions are: What are the crops to be grown? What is the area to be allocated to each crop? What are the irrigation levels to be applied for each crop through the whole growing season and therefore the water to be consumed? What is the expected yield of the each crop? And finally, what is the expected profit at the end of the cropping season?

The organization of this thesis is as follows: Chapter 2 provides a literature review. The problem is defined in Chapter 3. Chapter 4 discusses the first proposed model; namely the Dynamic Programming Model. The second proposed model; namely the Single Crop Model in explained in chapter 5. Chapter 6 discusses the integrated model; named the Multi-crop Model. The last model; named Irrigation Schedule Model is given in chapter 7. The last chapter is for conclusions and directions for future research.

## Chapter 2

---

### Literature Review

Many studies displayed in the literature address water deficit and other water-related concepts.

- [12] Conducted many experiments to determine the actual relations between yield and evapotranspiration, water purchased and applied, and field water supply. They developed a multiplicative formula for crop yield as a function of applied irrigation water. This formula is widely used in deficit irrigation.
- [6] Analyze experimental data on crop yield response to water and empirically derive yield response factors,  $ky_t$ , which are the sensitivity indices for water stress in the specified growth stage  $t$  of the crop.
- [7] Studied the relation between yield and water applied to determine the optimal amount of water that results in maximum yield.
- [9] Analyzed various water-related factors of crop production. They tried to show how deficit irrigation influence profit and summarized the situation in which deficit irrigation is less desirable. They also introduced a number of mathematical yield models relating crop evapotranspiration and total water available to relative yields.
- [13] Provided detailed procedures for estimating the expected yield per unit area for a given level of applied water as a fraction of the maximum yield that would be obtained at an optimal irrigation level.

A lot of effort has been devoted in the literature to build models that can generate optimal schedules of water releases and water allocation for crops under deficit irrigation. We will discuss some of these models.

- [14] Developed an integrated model based on seasonal inputs of reservoir inflow and rainfall in the irrigated area to determine the optimal reservoir release policies and irrigation allocations to multiple crops under water deficit. They considered the inflow and rainfall both to be stochastic. The model is conceptually made up of two modules. Module 1 is an intraseasonal allocation model to maximize the sum of relative yields of all crops, using linear programming (LP). This module considers the crops growth stages, soil types, soil moisture, and irrigation policy for each crop. Module 2 is a seasonal allocation model to derive the steady state reservoir operating policy using stochastic dynamic programming (SDP).

[14] Was an extension of some old work made by [13] that consider the same problem but under a deterministic framework. [13] Used a different approach in which they apply deterministic dynamic programming for the first module and a stochastic dynamic programming for the second module.

- [10] Proposed a two-stage decomposition approach to determine optimal seasonal multicrop irrigation water allocation and optimal stochastic intraseasonal (daily) irrigation scheduling. The first stage consists of two steps. In the first step, a relationship function between seasonal water availability and expected value of yields for each crop and for all possible initial soil moisture conditions is developed. In the second step, the optimal seasonal water allocation for each crop is defined so that total yield/ benefit from all crops is maximized. This is done using deterministic dynamic programming. The second stage uses the optimal seasonal water allocation resulting from the first stage and generates optimal irrigation policies for each crop on a daily basis, using stochastic dynamic programming.
- [5] Described an integrated decision support system (DSS) that aids the operation of a tank irrigation system in south India. The DSS includes data subsystem, a model subsystem, a knowledge base, and a user interface. The

model subsystem consists of two models, area allocation model, and water allocation model. The area allocation model provides optimal crop area to which irrigation can be ensured at all time periods (one month each) depending on a water year basis. It uses the linear programming technique to obtain the solution. The water allocation model uses dynamic programming to provide optimal weekly irrigation releases based on current inflow, irrigation demand, initial storage, and crop areas resulting from the first model.

- [11] Developed a general computer software package called ZERO1 using 0-1 integer programming to obtain optimal rotational schedules of irrigation using lateral canals. Their program divides the irrigation season into a number of rotational periods such that the demand of water in each period does not exceed the capacity on the canal.
- [15] Used quadratic programming to build an optimization approach that is able to determine optimal allocation of available water that maximizes the total expected yield, while ensuring equity between water users in the system. They calculated the expected yield using the additive form of the yield formula. Their model is to be applied primarily to run-of-river systems. They evaluated their approach through application into the irrigation system in the lower Ayung River Basin in Bali, and found that it gives good results.
- [3] Proposed a multi-stage decision model for the conjunctive use of ground and surface water with an artificial recharge, to determine the total amount of water to import, the fractions of imported water to allocate to irrigation and to the artificial recharge as well as the size of pumping at each period for some planning horizon. The model attempts to trade off the impact of partially not meeting the irrigation demand for a particular period against the impact of excessive groundwater pumping and not meeting future irrigation demands.
- [3] Developed a single period model for the conjunctive use of ground and surface water in a multi-reservoir system. In their model, they considered the case where the aquifer suffers from severe overdrafts. Consequently, they assumed that withdrawals from aquifer are to be established only under emergency cases. The inflow to the main reservoir and the demand of irrigation

water at local areas were considered as stochastic. They allowed deficit irrigation in the problem of maximizing expected total profit for the entire region.

- [4] Extended the model by Azaiez and Hariga (2001) to a multi-period model. They approached the problem as a chance-constrained model.
- [8] Proposed a multi-stage linear programming model for optimal cropping patterns under water deficits in dry regions. Their model identified both the total area and the irrigation levels allocated to a given selected crop taking into account the possible successors and predecessors of this crop. It also allowed choosing the same crop more than once, each time with a different irrigation level. First, the model optimized cropping patterns, for a given grower in the region of interest, and for a given stock of water available. Then, a global optimal cropping plan for the entire region was determined in order to permit optimal water allocation among growers of the region. They have found that the global problem is of a large scale. Therefore, they developed a decomposition algorithm to reduce the computational effort.

The model in [8] admits few limitations as pointed out below.

1. At the start of the plan, the model assumes that the entire land is available for exploitation. In practice however, part of the land might be still occupied by some crops and the corresponding land would be made available later after harvesting the crops in the occupied land.
2. The model assumes that the entire stock of water is available whenever needed. However, most often, the supply of water is made over time and it is seasonal. Furthermore, usually the supply is low when it is needed the most and vice-versa.
3. In practice, several crops have short growth periods and it is feasible to grow sequentially more than 2 crops in the same year (which is true for several types of vegetables). Thus, contenting with only summer and winter crops is limiting.
4. The model uses the same deficit irrigation level for all growth stages. However, some growth stages are usually more sensitive to water stress

than others (e.g., the flowering stage for many crops). Therefore, it is important to make the best water allocation in time so that sensitive periods will receive the least deficit and vice-versa.

5. The model can be used only by sophisticated users. Thus, a friendly user code can be very helpful in practice.
- [18] Developed a multi-stage decision model to evaluate the economic feasibility of on-farm impoundment-based supplemental irrigation under uncertainty, in the Blacklands region of Texas. The model included in particular crop mix selection. However, the model limited consideration to a single farm rather than to a region. The problem was formulated as a nonlinear mixed integer program.

Some effort has been devoted to review OR techniques in water management models. In this regard, [16] made a survey on dynamic programming models for water resource problems, and examined computational techniques which have been used to obtain solutions to these problems. [17] made an intensive literature review on mathematical models developed for water releases through reservoirs, including simulation. He classified the available methods as Linear Programming, Dynamic Programming, Nonlinear Programming, Simulation, and Combinations of the above methods.

He also discussed the capability of each method in successfully approaching real water problems.

# Chapter 3

---

## Problem Description

### 3.1 Scope

The current study will cope in particular with the limitations of the model given in [8]. The problem will explicitly use deficit irrigation over the different growth stages of a given crop. It will also manage several crops in the same land at different growth periods.

### 3.2 Problem Definition

The problem is specifically stated as follows. Given a piece of land with perhaps some crops still growing, which among candidate crops one should select to grow for the next cropping periods. The decision on the "best" cropping plan should account for many factors including land availability, the growth period, crop predecessors, water requirements, harvesting period, and economic and market considerations. The objective is to determine the optimal cropping pattern over the planning horizon (which is considered to be one year), in order to maximize the total expected profit.

The cropping pattern deals with determining the sequence of crops, the area to be allocated to each crop, the irrigation levels that will be applied to each crop at each growth stage, and therefore the water to be consumed in different periods (seasons) as well as over the entire cropping plan, and the irrigation schedule of each crop over each cropped area.

The year is assumed to consist of four different irrigation seasons according to water availability; each of these seasons has a certain starting date and certain amount of irrigation water. In fact, the Kingdom of Saudi Arabia is characterized by some very dry period during summer, where groundwater pumping is excessively applied to meet as much as possible irrigation demand. During winter, considerably less pumping occurs. Spring and fall are intermediate pumping seasons. When pumping is applied excessively, the water table significantly decreases reducing the pumping capacity. This occurs usually when crops need irrigation water the most. Thus, it is not obvious that full water requirement will be satisfied during the dry period.

Each crop will be associated with a number of possible irrigation levels; each irrigation level corresponds to certain water consumption and a certain expected yield. The Choice of irrigation levels for a given candidate crop is influenced by several factors; some of which are related to the location of the cropping area, others are related to the weather conditions for the region of interest, and others are related to the crop itself (mainly its behaviour towards applying different amounts of water).

This study will take all the information regarding the crop characteristics, the current land exploitation, the irrigation season information, in order to determine the optimal cropping pattern over the planning horizon.

A huge quantity of information has to be processed in the way towards the optimal cropping pattern. Grouping all this information in a single model will make it too complicated. Therefore, we propose to approach the problem sequentially by developing a number of models where the output of one model will serve as input to the next, until the optimal cropping plan is identified.

For this purpose, four models are envisaged, as given below:

1. Multi-crop Model
2. Single-crop Model
3. Dynamic programming Model
4. Irrigation allocation Model

Each of these models will be discussed in details to explain the input, output and the data flow inside the model. In general, each model may consist of a number of modules

that interact internally to give the final model's output. The relation between the models can be explained using the following chart.

As shown in the chart there is some input information that is classified in five basic categories:

1. Region information.
2. Previous crops information.
3. Candidate crops information.
4. Irrigation seasons information.
5. Weather data information.

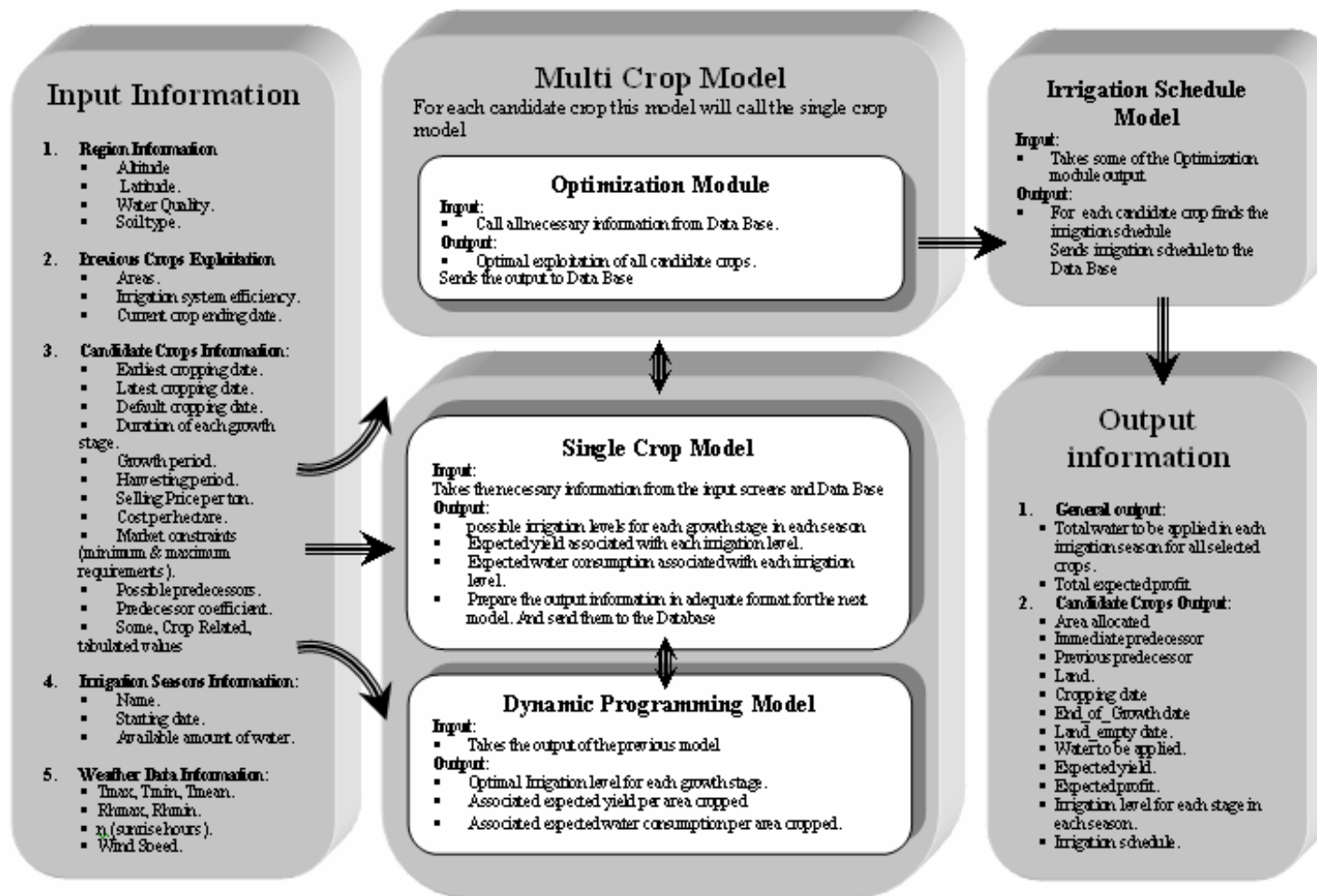


Figure 3.1: General Chart for the relation between Different models

## Chapter 4

---

# The Dynamic Programming Model

### 4.1. Overview of the model

As mentioned earlier, the crop life consists of a number of growth stages with the following characteristics of interest to us in this study.

- Each growth stage has its own water requirement. When full requirement is not satisfied, the crop is subjected to water stress leading to yield reduction.
- When water stress reaches a particular level (wilting point), the crop can not survive any further.
- It is possible for each growth stage to consider a number of possible irrigation levels that can be defined as the fraction of water applied to full water required. The minimum level should ensure crop survival.
- The sensitivity of the crop to water stress depends on the specific growth stage  $t$  and could be measured empirically by the yield response factor  $ky_t$ .

When the available amount of irrigation water is enough to satisfy full irrigation water requirement, the problem would be simply to allocate water in time and quantity in order to satisfy full requirement at each growth stage. This will normally result in maximum yield per unit area cropped (once all other input factors are taken at their optimum level).

If however the available amount of irrigation water does not satisfy full water requirement, then a decision should be made on the level of irrigation to apply for each growth stage in order to maximize the expected yield of the corresponding crop per unit area cropped.

The decision on the selected irrigation level for each growth stage should take into account the sensitivity of each growth stage to water deficit such that highly sensitive periods receive relatively high share of irrigation water and low sensitive periods receive low shares. Therefore, the proposed model will determine, for a given amount of irrigation water, the irrigation level that should be applied to each growth stage of the crop life in order to maximize the expected yield per unit cropped area.

#### **4.2. Objective of the model**

The objective of this model is to find the optimum water allocation over time of a given amount of irrigation water, for a given crop, in order to maximize the actual expected yield per unit cropped area. Equivalently, the model will rather maximize the ratio of actual expected yield to the maximum expected yield (which is obtained when full water requirement is satisfied over all growth stages of the crop).

#### **4.3. Inputs**

- The available amount of irrigation water
- The number of growth stages for the crop under consideration
- The possible irrigation levels for each growth stage.
- The water volume associated with each irrigation level in each growth stage.
- The sensitivity of each growth stage to water deficit (expressed through the yield response factor at growth stage  $t$ ,  $ky_t$ ).
- The maximum expected yield associated with full irrigation level.

#### **4.3. Outputs**

- The selected irrigation level for each growth stage of the crop life.
- The corresponding volume of water to be applied at each growth stage.

- The expected yield of the crop per unit area cropped.

#### 4.5. Notations

##### 4.5.1. Model Parameters:

- $n$  Total number of growth stages of the crop
- $t$ : Growth stage index,  $t=1, 2, \dots, n$
- $Y_m$ : Maximum expected yield that results from applying full irrigation level for each growth stage of the crop life. It can be estimated and it is considered as an input to the model.
- $Y_a$ : Expected actual yield
- $ky_t$ : yield response factor of the crop for growth stage  $t$ . It reflects the sensitivity of the growth stage to water deficit. Its value is empirically calculated and is available in the literature.
- $ET_{mt}$  Maximum evapotranspiration at growth stage  $t$
- $ET_{at}$ : Actual evapotranspiration at stage  $t$
- $WR_t$ : Quantity of water required at stage  $t$ .
- $WA_t$ : Quantity of water applied at stage  $t$ .
- $S_t$ : The state variable at each stage  $t$  consisting of the total amount of water applied over stages  $t, t+1, t+2, \dots, n$
- $J_t$  The total number of possible irrigation levels at growth stage  $t$
- $IL_j$ : The  $j^{th}$  irrigation level in the  $t^{th}$  growth stage of the crop life,  $j=1, 2, \dots, J_t$ .
- $\zeta(S_t)$ : The optimal contribution of stages  $t$  to  $n$  to the ratio of actual to maximum yield to be used in the recursive equation of the dynamic program, DP.

#### 4.6. The elements of the dynamic program

1. Stages: 1 to  $n$  corresponding to the crop growth stages.

2. Alternatives at each stage: the possible irrigation levels at each stage that can vary from the minimum to maximum (100%) irrigation level. Minimum and maximum water requirement of the crop are obtained as outputs of the yield model discussed in Chapter 5. The candidate irrigation levels are also specified in the yield model.
3. State variables:  $S_t$  representing the total amount of water available for growth periods  $t, t+1, t+2, \dots, n$ .
4. The measure of performance  $\zeta(S_t)$  for the recursive equation of the DP.

The measure of performance in the DP is the ratio of the expected to the maximum yield. This ratio can be calculated using various methods. In this study, we focus on two different approaches, which are the most commonly used. The first is referred to as the multiplicative form, and the second as the additive form. Further discussion will be offered below. The following development will consider the multiplicative form. The additive form will have a similar procedure of calculation and will be briefly discussed later.

#### 4.7. The multiplicative form

The fraction of the expected to maximum yield can be calculated by the following formula, which was widely used in the literature [12], [3], [8].

$$\frac{Y_a}{Y_m} = \prod_{t=1}^{n_c} \left( 1 - k y_t \left( 1 - \frac{ET_{at}}{ET_{mt}} \right) \right), \quad (4.1)$$

This formula has been proven to give a good estimation of the yield for most crops [6]. The maximum and the actual evapotranspiration can be represented by the ratio of water applied to that of water required. It is plausible to assume that the ratio of actual to maximum crop evapotranspiration is the same as the ratio of water applied to water required [15]. That is,

$$\frac{ET_{at}}{ET_{mt}} = \frac{WA_t}{WR_t} \quad (4.2)$$

Therefore, the ratio of actual to maximum evapotranspiration will be referred to in this study as the irrigation level.

$$IL_t = \frac{ET_{at}}{ET_{mt}} = \frac{WA_t}{WR_t} \quad (4.3)$$

It should be noted that each stage may have a number of candidate irrigation levels that may differ from one stage to the other for the same crop as well as for different crops. Therefore, we will add another index to the irrigation level notation to designate the irrigation level in the stage under consideration. The new notation will be:

$IL_{ij}$  : The  $j^{th}$  irrigation level in the  $i^{th}$  growth stage of the crop,  $j=1, 2, \dots, J_i$

The irrigation level at each growth stage will vary from a minimum value ( $IL_{i1}$ ) (below which the crop would not survive) to full irrigation level ( $IL_{iJ_i}$ ) (in which case the water applied is equal to water required).

Substituting (4.3) in (4.1) yields the following formula:

$$\frac{Y_a}{Y_m} = \prod_{t=1}^{n_c} (1 - ky_t (1 - IL_{ij})), \quad (4.4)$$

According to the above formula, the fraction of the expected to maximum yield, which is the suggested measure of performance in this particular model, will differ according to the selected irrigation levels over each growth stage of the crop life. Therefore, the decision variable would be the irrigation level at each growth stage.

The recursive equation of the proposed dynamic program will be derived from equation (4.4) as follows:

The contribution to the measure of performance at a single stage  $t$  (which is the ratio of actual to maximum yield) can be written in the following form:

$$(1 - ky_t(1 - IL_{tj})) \quad (4.5)$$

Therefore, for the last stage  $t = n$ , this contribution is optimized as follows:

$$\zeta(S_n) = \max_{IL_{n1}, IL_{n2}, \dots, IL_{nj}} [1 - ky_t \times (1 - IL_{nj})] \quad (4.6)$$

Subject to:

$$S_n \geq WR_n \times IL_{nj} \quad (4.7)$$

Equation (4.6) will select the best irrigation level for the last stage while not exceeding the available amount  $S_n$  of irrigation water for that stage.

Since we are using the multiplicative form of the measure of performance the contribution of previous stage ( $t=n-1$ ) is calculated by equation (4.5) for each irrigation level and multiplied by the contribution of the current stage ( $n$ ). Therefore, the recursive formula for any stage ( $t < n$ ) is given by

$$\zeta(S_t) = \max_{IL_{t1}, IL_{t2}, \dots, IL_{tj}} \{ [1 - ky_t \times (1 - IL_{tj})] \times \zeta(S_{t+1}) \} \quad (4.8)$$

Subject to:

$$S_t \geq WR_t \times IL_{tj} + S_{t+1}, \text{ for all } j \quad (4.9)$$

When  $t=1$ , the recursive formula given by (4.8) and (4.9) will provide the optimal ratio of actual to maximum expected yield, as given in (4.10) and (4.11) below.

$$\frac{Y_a}{Y_m} = \zeta(S_1) = \max_{IL_{11}, IL_{12}, \dots, IL_{1j}} \{ [1 - ky_1 \times (1 - IL_{1j})] \times \zeta(S_2) \} \quad (4.10)$$

Subject to:

$$S_1 \geq WR_1 \times IL_{1j} + S_2 \quad (4.11)$$

Moreover, the optimal water allocation over time to the crop can be obtained by obtaining successively the selected irrigation level  $IL_{tj}$  at each stage  $t$  of the DP.

#### 4.8. The additive form

The additive form of the objective function (representing the decrease in yield when applying deficit irrigation) was also used in the literature (i.e. [14],[5]). This form also gives a good estimation of the ratio of actual to maximum yield. The formula is given by:

$$\frac{Y_a}{Y_m} = 1 - \sum_{t=1}^n Ky_t \left( 1 - \frac{ET_{at}}{ET_{mt}} \right) \quad (4.14)$$

When using the additive version of the objective function, the recursive formula will slightly change to take the following form:

$$\zeta(S_t) = \text{Min}_{IL_{t_1}, IL_{t_2}, \dots, IL_{t_t}} [ky_t \times (1 - IL_{ij})] + \zeta \left( \sum_{i=t+1}^n WR_i \times IL_{ij} \right) \quad (4.15)$$

Subject to:

$$S_t \geq WR_t \times IL_{ij} + S_{t+1} \quad (4.16)$$

#### 4.9. Illustrations

For the remaining of the thesis, we will only use the multiplicative form, which is the most popular formula and which is appropriate with most crops. However, the additive form could also be used simply by changing the recursive formulae, as explained above.

Back to the previous recursive formulae (4.10) - (4.15), when the available amount of irrigation water can satisfy full water requirement of the crop through all growth stages, then the dynamic program will select the full irrigation level at each stage. This leads to a ratio of actual to maximum yield of 1. That is, the maximum yield will be obtained. Otherwise, the dynamic program will determine the irrigation levels to be used in each stage to maximize the expected actual yield.

In the dynamic program, break-even cases are treated so that the solution will allocate the highest share of irrigation water to the most sensitive growth stage (having the *highest*  $ky_t$ ).

The procedure is as follows:

- Starting from the first growth stage, rank the growth stages in decreasing order with respect to yield response factors.
- Compare the irrigation levels allocated to the first ranked growth stage.
- The solution that allocates the higher irrigation level will be considered as the selected solution.
- If the irrigation level in both sets were equal, then the model will consider the next growth stage in the ranked stages and so on.

The following example will illustrate the process for treating break-even cases.

#### Example 4.1

Considering a crop that has five growth stages, the yield response factor for each stage is shown in the first row of table 4.1. The second and third rows represent two candidate solutions of irrigation levels and the same resulting value of the objective function. Both solutions allocate the same highest share of irrigation water to the most sensitive stage (95% of requirement for stage 3). However, the first solution allocates 90% of water requirement at stage 4 (which is the second most sensitive stage to water stress) versus 85% only allocated by the second solution. Therefore, the model will select the first solution.

	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5	Objective function
$ky_t$	0.2	0.3	0.65	0.55	0.2	
First solution	80%	83%	95%	90%	80%	92%
Second Solution	82%	85%	95%	85%	82%	92%

**Table 4.1: Break-even example**

It should be noted that the dynamic program is developed to solve the problem for any number of stages. However, in the application, most crops have up to five growth stages (e.g., Vedula & Mujumdar 1992)).

The procedure in the dynamic program will be illustrated by the following example.

### Example 4.2

Considering green Maize that has four growth stages. The values of the yield response factor for the different growth stages are given in the following table:

Stage 1	Stage 2	Stage 3	Stage 4
Establishment	Vegetative	Flowering	Yield formation
0.4	1.5	0.5	0.2

**Table 4.2: Growth stages and the corresponding yield response factor of Green Maize[6]**

Note that the dynamic program considers the maximum yield and the full water requirements as known and treats them as inputs to the model. In order to use some informative values of these inputs, we will actually estimate them according to a procedure explained in the next chapter. The details on possible irrigation levels and associated water consumption for each growth stage are given in the following table (they are the output of the single crop model discussed later in chapter 5):

S. No.	Stage 1		Stage 2		Stage 3		Stage 4	
	IL <sub>1</sub>	WR <sub>1</sub> (m <sup>3</sup> /ha)	IL <sub>2</sub>	WR <sub>2</sub> (m <sup>3</sup> /ha)	IL <sub>3</sub>	WR <sub>3</sub> (m <sup>3</sup> /ha)	IL <sub>4</sub>	WR <sub>4</sub> (m <sup>3</sup> /ha)
1	0.5	199	0.5	949	0.5	2094	0.5	1196
2	0.55	219	0.55	1044	0.55	2303	0.55	1316
3	0.6	239	0.6	1139	0.6	2513	0.6	1435
4	0.65	259	0.65	1234	0.65	2722	0.65	1555
5	0.7	279	0.7	1329	0.7	2931	0.7	1674
6	0.75	299	0.75	1424	0.75	3141	0.75	1794
7	0.8	319	0.8	1519	0.8	3350	0.8	1914
8	0.85	339	0.85	1613	0.85	3559	0.85	2033
9	0.9	359	0.9	1708	0.9	3769	0.9	2153
10	0.95	379	0.95	1803	0.95	3978	0.95	2273
11	1	399	1	1898	1	4188	1	2392

**Table 4.3: Possible irrigation levels and associated water requirement for each growth stage of the Green Maize life**

The maximum yield in case of applying full water requirement for each growth stage of the crop life is 20,54 Ton/Ha (calculated by the single crop model).

The dynamic program output for different amounts of available water is given in the following table:

Available Water (m <sup>3</sup> /Ha)	Optimal irrigation levels				Expected yield (Ton/Ha)
	Stage 1	Stage 2	Stage 3	Stage 4	
4800	0.65	0.65	0.5	0.5	5.72
5000	0.7	0.75	0.5	0.5	7.80
6000	0.95	1	0.6	0.5	14.63
6000	1	1	0.7	0.5	15.87
7000	1	1	0.8	0.55	16.98
8000	1	1	1	0.6	19.08
9000	1	1	1	1	20.74

**Table 4.4: Dynamic Program output for Green Maize according to different amounts of irrigation water.**

The maximum and minimum water requirement for green maize are 8876m<sup>3</sup>/ Ha, 4438m<sup>3</sup>/Ha respectively (calculated by the single crop model).

Table 4.4 shows that when the amount of water is not sufficient to satisfy full water requirement, the model allocates the water to the stages according to the sensitivity of the growth stage, which is measured as mentioned earlier by the yield response factor. Therefore, if the amount of water is 4800 m<sup>3</sup>/ Ha, then stage 2 will have the highest irrigation level. This is justified from the fact that this stage is the most sensitive growth stage of the crop.

It is worth noting that the selected irrigation level for the first growth stage is higher than that of the third growth stage, although stage 3 is more sensitive than stage 1. This however does not ruin the assumption but could be justified from the fact that the improvements in irrigation levels are made according water availability at a 15% increment. In our particular case, stage 1 needs only 20 m<sup>3</sup> of water per ha to achieve

the next irrigation level, while stage 2 requires  $m^3$  and stage 3 requires  $210 m^3$  to reach the same level of improvement. After satisfying the minimum requirement for all stages, the DP allocated 65% level to the most sensitive stage (i.e., stage 2). The remaining amount of water is  $77 m^3$ , is enough to increment the third most sensitive growth stage (stage 1) by 15%.

## Chapter 5

---

# The Single Crop Model

### 5.1. Overview of the model

The model tries to maximize the total expected yield for a single crop over a certain land under deficit irrigation. In fact, if water supply is limited, satisfying full water requirement may be realized at the expense of leaving some of the land uncropped. This would maximize the yield per unit area cropped but not necessarily the total expected yield over the entire available land. Deficit irrigation may however reduce the yield per unit area cropped while increasing the irrigated area. Therefore, the loss in yield per unit area cropped can be overstepped by yield coming out from the additional irrigated area. As a result, the total expected yield may increase under deficit irrigation while using the same amount of irrigation water.

The model will determine the land to be used and the irrigation levels to be applied for each growth stage of the crop life in order to maximize the total expected yield of a single crop over some given land. The model will consider two different situations regarding the area of the land to be cropped; namely the case of unconstrained area and the case of limited area. The first situation is motivated by the fact that many farming companies in Saudi Arabia are currently using only a small portion of their available land because of the restrictions imposed by the government on their market share of wheat. The second situation reflects the case of small farmers that normally have a limited cropping area.

## 5.2. Objective of the Model:

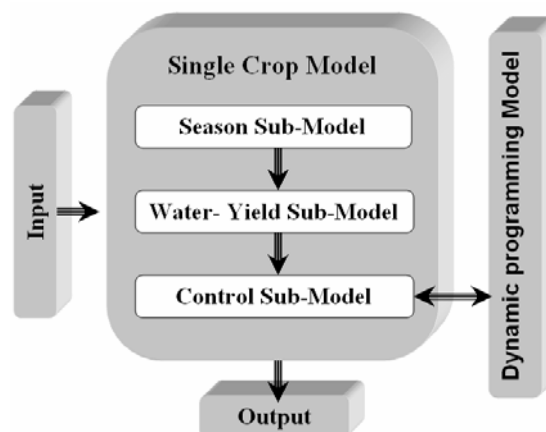
This model determines the optimal allocation of limited irrigation water in time and space in order to maximize the total expected yield of a single crop over some available land. In other words, it determines the area to be cropped, and the levels of irrigation to be applied to each growth stage. The problem is further complicated by the seasonal supply of irrigation water. This constrains the ability to allocate high shares of irrigation water during highly sensitive growth stages to water stress. Moreover, from modeling point of view, one should account in allocating water for both the seasonal supply of water and the crop growth period for situations where a given period falls in two different supply seasons with different pumping capacities. It will also serve in providing different combinations of irrigation levels as inputs to the multi-crop model, discussed in the next chapter..

## 5.3. Methodology:

The model will use three sub-models. These sub-models will interact internally to reach the final output of the model. The proposed sub-models are:

1. Season sub-model
2. Water-Yield sub-model
3. Control sub-model

The following chart briefly represents the relation among the different sub-models.



**Figure 5.1: Brief Representation of the Information Flow inside the Single Crop Model.**

## 5.4. Season Sub-model

### 5.4.1. Overview

In most regions of the kingdom of Saudi Arabia, groundwater is practically the only source of irrigation water. Farmers pump irrigation water from aquifers. During hot periods, water demand is high and consequently excessive pumping is performed. This results in decreasing the water table in the particular region. Usually, it takes few hours until the water table gradually increases to stabilize at some equilibrium level by having groundwater replenishment from other regions that observe less pumping. This affects negatively the withdrawal capacity of groundwater. During wet periods the demand of groundwater decreases and withdrawal capacity is at its peak. Therefore, the irrigation water is not available in the same quantity throughout the year. The sub-model considers this fact and assumes that the year consists of four different irrigation seasons according to water availability. The assumed four irrigation seasons are as follows.

- 1- Low consumption season.
- 2- Intermediate season (Low to High consumption).
- 3- High consumption season
- 4- Intermediate season (High to Low consumption).

Each of these seasons is associated to a certain period over the year and a certain pumping capacity (pumping hours per day leading to some volume of water per season).

As discussed earlier, each crop has a cropping period and several growth stages, each with certain duration. The season sub-model will take this information regarding the given crop and the irrigation seasons information to determine the number of growth stages that fall in each irrigation season.

### 5.4.2. Objective

The objective is to determine the growth stages of a given crop  $c$  that exist over each irrigation season.

### 5.4.3. Inputs

The input is categorized in two classes:

1. Irrigation seasons information:
  - Starting date of each irrigation season
  - Duration of each irrigation season
  - Pumping capacity during the season
2. Crop information:
  - number of crop growth stages
  - starting date of each growth stage
  - Duration in days of each growth stage.

### 5.4.4. Outputs

- Number of growth stages that exist over each irrigation season.
- Duration in days of each growth stage per season

### 5.4.5. Notations

- $s$ : Season index  $s=1, 2, 3, 4$ , the model assumes four irrigation seasons
- $SeSDt_s$ : Starting date of season  $s$
- $SeEDt_s$ : Ending date of season  $s$  (which is  $SeSDt_{s+1} - 1$ )
- $StSDt_t$ : Starting Date of stage  $t$
- $StEDt_t$ : Ending Date of stage  $t$  (which is  $StSDt_{t+1} - 1$ )
- $D_t$ : Duration in days of stage  $t$

$D_{t,s}$  : Duration of stage  $t$  in season  $s$

$fn_c$  : Final number of growth stage for crop  $c$

#### 5.4.6. The procedure

It is possible for a particular growth stage of the crop to lie in more than one irrigation season. In that case, the growth stage will be divided into two sub-stages, having the same crop growth-related characteristics including the yield response factor. The duration of each sub-stage depends on the number of days that exist over each irrigation season.

The following simple procedure will determine the growth sub-stages that exist over each irrigation season and their duration.

The initial conditions are:

Start the procedure for the first season ( $s=1$ )

For the first growth stage ( $t=1$ )

The final number of the growth stages is the same as the original number of growth stages ( $fn_c = n_c$ )

*1. If the starting date of growth stage  $t$  is less than the starting date of irrigation season  $s$*

*And at the same time*

*The Starting date of stage  $t$  is less than the ending date of irrigation season  $s$*

*And at the same time*

*The ending date of stage  $t$  is greater than the ending date of irrigation season  $s$*

*If  $StSD_t \geq SeSD_s$  and  $StSD_t < SeED_s$  and  $StED_t > SeED_s$*

*Then*

*Stage ( $t$ ) will be treated as two different stages. The first stage is in season  $s$  and the other stage is in season  $s+1$*

*The duration of the two stages is as follows:*

*Duration of stage t in season(s) =*

*The starting date of irrigation season(s) – starting date of the growth stage (t)*

$$D_{t,s} = SeSD_s - StSD_t$$

*The duration of growth stage (t) in season(s+1) =*

*The duration of growth stage (t) – the duration of the same growth stage in season(s)*

$$D_{t,s+1} = D_t - D_{t,s}$$

*Add one to the final number of growth stages*

$$fn_c = fn_c + 1$$

*Go to the next growth stage.*

$$t = t + 1$$

*If there is such growth stage (the previous growth stage is not the last stage)*

$$\text{If } t \leq n_c$$

*Then*

*Repeat the previous calculations*

*Go to 1*

*Else*

*No more stages for calculation.*

*Go to 3*

*2. Else if the starting date of growth stage (t) is greater than or equal to starting date of irrigation season (s)*

*And at the same time*

*The ending date of growth stage (t) is less than the ending date of irrigation season (s).*

$$\text{Else If } StSD_t \geq SeSD_s \text{ and } StED_t \leq SeED_s$$

*Then stage t exists totally over season (s)*

*The duration of growth stage (t) over irrigation season(s) is the same duration of the growth stage (t)*

$$D_{t,s} = D_t$$

*Go to the next growth stage.*

$$t = t + 1$$

*If there is such a growth stage (the previous growth stage is not the last stage)*

$$\text{If } t \leq n_c$$

*Then*

*Repeat the previous calculations*

*Go to 1*

*Else*

*No more stages for calculation.*

*Go to 3*

*Else*

*Go for the next irrigation season*

$$s = s + 1$$

*If there is such irrigation season (the previous irrigation season is not the last one)*

$$\text{If } s \leq 4$$

*Then*

*Repeat the previous calculations*

*Go to 1*

*Else*

*No more irrigation season for calculations.*

*Go to 3*

*3. Number of growth stage for the corresponding crop is the calculated final number of growth stages denoted by.*

$$n_c = fn_c$$

*Got to 4*

*4. End*

The output of this season sub-model will be used as input to the yield sub-model below.

### Example 5.1

Consider the hypothetical seasons' information given in Table 5.1. The process goes as follows.

Season	Starting Date	Total Available water (m <sup>3</sup> )
1	Oct 1	200,000
2	Dec 1	300,000
3	March 15	140,000
4	May 15	80,000

**Table 5.1: Seasons information**

We will consider Green maize for our example, which has the following growth stage information (Doorenbos and Kassam, 1979).

Green Maize	Stage 1	Stage 2	Stage 3	Stage 4
Duration	20	40	50	30
Starting date	Mar 1	Mar 21	Apr 30	Jun 19

**Table 5.2: Starting date and durations of each growth stage of Green Maize**

The following table summarizes the result of the season sub-model according to the proposed procedure:

Season	Total Available water (m <sup>3</sup> )	Sub-stages	Duration (days)
1	200,000	--	--
2	300,000	Sub-stage 1,1	14
3	140,000	Sub-stage 1,2	6
		Stage 2	40
		Sub-Stage 3,1	15
4	80000	Sub-Stage 3,2	35
		Stage 4	30

**Table 5.3: Output example for the Season Sub Model**

## **5.5 Water-Yield Sub-Model**

### **5.5.1. Overview**

It is responsible for estimating water requirement and the corresponding crop yield. It uses many inputs related to climatic conditions, geographic information, soil characteristics, and water quality at the region of interest. In this thesis, we develop a computer code, in order to relate automatically the different existing formulae using the different inputs in order to generate water requirements and maximum yield for each crop of interest. The sub-model assumes that all other input factors to crop yield (such as pesticides, know-how, seeds ... etc.) are kept at their optimal levels.

### **5.5.2. Objective**

This sub-model provides water requirements and the maximum yield when full water requirements are satisfied.

### **5.5.3. Inputs**

The inputs to the model can be categorized in three different categories:

1. Crop inputs:
  - Cropping date
  - Duration of the growth stages of the crop life.
2. Planting region inputs:
  - Altitude
  - Latitude
  - Soil texture
3. Whether data inputs

- Maximum, minimum, and average temperature during each growth stage
- Maximum and minimum relative humidity during each growth stage
- Average sunrise hours during each growth stage
- Wind speed during each growth stage
- Some tabulated values related to the previous inputs

#### 5.5.4. Outputs

- water consumption in each growth stage
- Maximum yield per hectare

The sub-model is proposed by [1]. The contribution of the current study is in coding the proposed sub-model using programming language Visual Basic 6, and validating it in order to implement it with other proposed models in the current study. A detailed flow chart representing the data flow and the calculation equations are attached in Appendix A. We content here by showing the type of output we obtain from the submodel through the following example.

#### Example 5.2

The following table represents an output of the Water-Yield Sub-model according to some input for which the details are omitted for convenience. However, interested readers are referred to Appendix A where they can find sufficient details.

Crop Name	Maximum yield (Ton/Ha)	Net water requirements (m3/Ha)	Leaching Requirement	Irrigation Efficiency %	Total water requirements (m3/Ha)
Green Maize	20.74	13064	0.1	70	20737
Onion (Oct 15)	23.64	3146	0.1	70	4994
Potato (Jan 1)	31.92	6112	0.1	70	9702
Tomato (Jan 15)	14.59	5888	0.1	70	9346
Wheat (Dec 15)	20.95	4007	0.1	70	6360

Table 5.4: Output summary for different crops according to the yield sub-model

The output in table 5.4 is compared with previously published tables by [19] for water requirements of different crops in Saudi Arabia, and it shows that the proposed yield model gives a good estimate of water requirements for these crops.

When considering the output taken from the “Season sub-model” Table 5.3, which is the modified number of growth stages according to the existence of the growth stages over the irrigation seasons, the output of the Water-yield sub-model for Green Maize will be as follows:

	Season 1	Season 2		Season 3		
Duration (Days)	Stage 1		Stage 2	Stage 3		Stage 4
	20			50		
	Stage 1,1	Stage 1,2	Stage 3,1	Stage 3,2		
	14	6	40	15	35	30
Daily Water Requirement (m <sup>3</sup> /Ha)	20	20	47	84	84	80
Maximum Yield (Ton)			20.74			

Table 5.5: Output summary for Green maize  
According to the Season sub-model and the Water-yield sub-model

## **5.6 Control Sub-Model**

### **5.6.1. Overview**

This sub-model controls the optimization procedure by performing the required calculations to determine the land constraints. It also controls the interfacing procedure with the dynamic programming model according to the number of growth stages available in each irrigation season, in order to maximize the total expected yield of a single crop over the region of interest.

### **5.6.2. Objective**

Maximize the total expected yield of a given crop over a given region, under one of the following two situations.

- The area of the cropping region is unconstrained
- The area of the cropping region is constrained

The submodel takes into account water availability in each irrigation season

### **5.6.3. Inputs**

- Available amount of irrigation water in each irrigation season.
- Number of growth sub-stages that exist over each irrigation season.
- Duration in days of each growth sub-stage
- Maximum yield per hectare
- Water requirement per hectare for each growth stage (full water requirement)
- Land constraints if any

Note that some of the inputs are supplied from the season and the water-yield submodels.

#### 5.6.4. Outputs

- Possible irrigation levels for each growth stage
- Water requirements associated with each irrigation level of each growth stage
- Total expected yield
- Land to be used
- Selected irrigation level for each growth stage of the crop life
- Water consumption associated with each selected irrigation level
- Total amount of water to be consumed during each irrigation season

In addition to the previous output, the model will serve in defining a set of combinations of irrigation levels which will be used by the multi-crop model. The procedure of getting this set will be discussed at the end of the current sub-model

#### 5.6.5. Notations

##### a. Parameters

- $WC_{ij}$  : Water consumption during growth stage  $t$  if irrigation level  $j$  is applied
- $FWR_t$  : Full water requirement associated to 100% irrigation level
- $OF_s$  : The value of the objective function at irrigation season  $s$
- $n_s$  : Number of growth stages that exist over irrigation season  $s$
- $ky_t$  : yield response factor
- $S_c$  : Number of irrigation seasons during crop life
- $WR \min_s$  : Minimum water requirement of the crop during irrigation season  $s$
- $WR \max_{t_s}$  : Maximum water requirement for stage  $t$  during irrigation season  $s$
- $WR \max_s$  : Maximum water requirement of the crop during irrigation season  $s$
- $WR \max_{t_s}$  : Maximum water requirement for stage  $t$  during irrigation season  $s$
- $Land \max_s$  : Maximum land that can be utilized through irrigation season  $s$

$Land\ min_s$  : Minimum land that can be utilized through irrigation season  $s$

$AW_s$  : Available water at irrigation season  $s$

$ApW_s$  : Applied water at irrigation season  $s$

$AWh_s$  : Available irrigation water per hectare during irrigation season  $s$

$ApWh_s$  : Applied irrigation water per hectare during irrigation season  $s$

$Y_m$  : Maximum yield (when applying full irrigation level at each growth stage)

#### **b. Decision variables**

*Land Used*: the area of land allocated to the crop

$IL_{jt}$  : The selected  $j^{th}$  irrigation level for each growth stage  $t$  of the crop life

$Y_e$  : Expected yield per hectare

$TY_e$  : Total expected yield of the crop of interest over the entire land

#### **5.6.6. Methodology**

The optimization procedure is performed at each season of the crop life (separately from the other seasons). In fact, each season has some amount of available irrigation water that is not related to the other irrigation seasons. The contribution to the yield at each season as expressed by the multiplicative formula of the DP is multiplied by the results of the other seasons to get the final expected yield. The optimization procedure is performed through interfacing with the dynamic programming model.

#### **5.6.7. Formulation**

The yield sub-model, as mentioned earlier, calculates full water requirement for each growth stage (associated with 100% irrigation level). The minimum possible irrigation level for each growth stage is assumed to be 50% of the full water requirements (corresponding to the wilting point of the crop, i.e., the level at which the crop survival is no longer possible). Other irrigation levels are generated by discretizing

the difference between the maximum and minimum irrigation levels with a suitable increment (taken to be 5% in this study). Therefore, each growth stage will have 11 possible irrigation levels (50%, 55%, 60%, ..., 95%, 100%). The water consumption associated with a given irrigation level is simply calculated as follows:

$$WC_{ij} = IL_{ij} \times FWR_t \quad (5.1)$$

As mentioned earlier, the crop life may extend over more than one irrigation season (each having some specific amount of irrigation water). Equation (4.4) states that the fraction of the expected to maximum yield of any crop is a function of water applied at each growth stage of the crop life. Since each irrigation season has its own water availability, the fraction of the expected to maximum yield for each season (which is defined as the objective function), can be maximized for each season separately from the other seasons,

$$OF_s = \prod_{t=1}^{n_s} (1 - ky_t (1 - IL_{ij})) \quad (5.2)$$

The fraction of the expected to maximum yield, of any crop, can therefore, be the multiplication of the optimized value of the objective function over the irrigation seasons associated to the crop life

$$\frac{Y_e}{Y_m} = \prod_{s=1}^{S_c} OF_s \quad (5.3)$$

From the above equation (5.2), the expected yield per unit cropped area can be calculated as follows:

$$Y_e = Y_m \times \prod_{s=1}^{S_c} OF_s \quad (5.4)$$

The total expected yield of the entire cropped area is calculated as follows:

$$TY_e = LandUsed \times Y_e = LandUsed \times Y_m \times \prod_{s=1}^{S_c} OF_s \quad (5.5)$$

Substituting equation (5.1) in equation (5.4) we get the final equation of the total expected yield:

$$TY_e = Y_m \times LandUsed \times \prod_{s=1}^{S_c} \prod_{t=1}^{n_s} (1 - ky_t (1 - IL_{tj})) \quad (5.6)$$

Equation (5.5) shows that the value of the total expected yield of any crop over a certain region is a function of the area of land used for the crop, and the irrigation level applied to each growth stage of the crop life. Therefore, the model will try to determine the area of land to be cropped, and the best irrigation levels to be applied for each growth stage in order to maximize the total expected yield.

Equation (5.5) is considered as the objective function to be maximized under two types of constraints.

- The first type is related to the available amount of water in each season.
- The second is related to the area of land that can be cropped.

The first constraint is written as follows:

$$WAp_s \leq WA_s \quad (5.7)$$

The value of the water applied in each season is calculated as follows:

$$WAp_s = ApWh_s \times LandUsed \quad (5.8)$$

From the above equation (5.7)

$$ApWh_s = \frac{WAp_s}{LandUsed} \quad (5.9)$$

In the same sense, water available per hectare per season is calculated as follows:

$$AWh_s = \frac{WA_s}{LandUsed} \quad (5.10)$$

Substituting equations (5.7) and (5.8) in equation (5.6), we get

$$ApWh_s \leq AWh_s \quad (5.11)$$

That is, the water applied per hectare per season depends on the selected irrigation level for each growth stage that lies over the season. Therefore, for each value of the land used, the available amount of irrigation water per hectare will differ, and the selected irrigation level for each growth stage of the crop over the corresponding season will change.

The optimal allocation of the available amount of irrigation water per hectare per season can be determined using the DP model discussed in Chapter 4.

The second constraint for the optimization procedure is the land used. The following discussion will consider determining the range of selecting the (*Land Used*)

In general, the area of land that can be used in each irrigation season is calculated as follows:

$$Land_s = \frac{WA_s}{WR_s} \quad (5.12)$$

Water requirement during each season depends on the water requirements of the different growth stages that lie over the season as follows:

$$WR_s = \sum_{t=1}^{n_s} WR_t \quad (5.13)$$

Water requirement at each growth stage vary from a minimum to a maximum value. Maximum water requirement is associated with full irrigation level. Minimum water requirement is associated with the minimum possible irrigation level. Therefore, water applied during the season will vary from a minimum to a maximum value according to the selected irrigation level for each growth stage that lies over the season. The maximum water requirement during the irrigation season is associated with selecting the full irrigation level for each growth stage of the crop, and similarly, the minimum water requirement is associated with the minimum irrigation level at each growth stage that lies on the season. That is,

$$WR \max_s = \sum_{t=1}^{n_s} WR \max_t \quad (5.14)$$

$$WR \min_s = \sum_{t=1}^{n_s} WR \min_t \quad (5.15)$$

From equation (5.11) we find:

$$Land \max_s = \frac{WA_s}{WR \min_s} \quad (5.16)$$

$$Land \min_s = \frac{WA_s}{WR \max_s} \quad (5.17)$$

Maximum and minimum land that can be utilized through each irrigation season can differ from one season to the other, since that the water availability in each irrigation season and/or the crop water requirement during each irrigation season could differ from one season to the other. Because the the area to be used at all seasons must coincide, the smallest among  $Land \max_s$ , for all  $s=1, 2, S_c$ , will be considered as a land constraint independently of the initial setting of the problem that may or may not account for a land constraint.

When the region of interest has no initial land constraints, the only constraint regarding the land will be related to the smallest  $Land\ max_s$ . But if the region of interest has already an initial land constraint, then model will take the minimum of the initial land constraint and the calculated smallest  $Land\ max_s$ .

On the other hand, the smallest among the values of the  $Land\ min_s$  will be considered as the minimum land that can be used.

As a summary of the above discussion, the model maximizes the value of the total expected yield given in equation (5.5) as follows:

Objective function:

$$\text{Maximize } TY_e$$

Subject to

$$Land\ Used \leq Land\ Constraint$$

$$Land\ Used \geq Land\ min_s$$

$$WA_{p_s} \leq WA_s$$

In the above model,  $Land\ Constraint$  will simply be the minimum of the initial land constraint and the calculated smallest  $Land\ max_s$ , or simply the calculated smallest  $Land\ max_s$ , according to whether the model is initially constrained or unconstrained in land, respectively.

In order to optimize the total expected yield, the model constructs a loop of calculations starting from  $Land\ constraint$  up to smallest  $Land\ min_s$  with a reasonable decrement. The decrement (calculation step) depends on the required precision of the land value. We propose the decrement to be 0.5 hectare. This value can be changed without any change on the optimization procedure; the only effect will be on the computation time that will increase by decreasing the decrement value.

Each value of the  $Land$  will reflect a certain amount of water available per hectare through each irrigation season such as:

$$AWh_s = \frac{AW_s}{Land} \quad s=1, 2 \dots Sc \quad (5.18)$$

the control submodel interfaces with the DP model, sends the values of ( $AWh_s$ ) in each season along with the number of growth stages that lie on each season in order to find the best allocation of the amount of irrigation water over all the growth stages, and the value of the objective function for each irrigation season. Then, it calculates the total expected yield for each value of the land used. The maximum total expected yield through all values of  $Land$  used between  $Land \min_s$  and  $Land \text{ constraint}$  will be the final output of the calculation.

As mentioned earlier, each season may contain a number of growth stages. If one growth stage lies over two irrigation seasons, then it will be treated as two different growth sub-stages having the same yield response factor and other stage-related factors. The DP will allocate at each sub-stage an irrigation level. These irrigation levels would naturally differ. In the proposed control sub-model, we use a weighted average of the irrigation level to the entire growth stage proportionally to the length of each of its sub-stages.

The optimization procedure can be summarized as follows:

$$Land = Land \min_s$$

$$MaxTY_e = 0$$

1- For each  $s=1, 2 \dots Sc$  calculates:

$$AWh_s = \frac{AW_s}{Land}$$

2- for each irrigation season interface with the dynamic programming model

Send the following information:

$$AWh_s,$$

Number of growth stages  $n_s$ ,

$Ky_{t,}$

*Possible irrigation level for each growth stage,*

*Water consumption per hectare associated with each irrigation*

*level*

*The out put is:*

*Value of the objective function for each season*

*Selected irrigation level for each growth stage*

*Water consumption per hectare*

3- *Calculate  $TY_e$  using equation (5.5) or (5.6)*

4- *If  $MaxTY_e < TY_e$  then  $MaxTY_e = TY_e$*

5- *Land = Land + increment*

*If Land  $\geq$  Land<sub>smin</sub> go to 1 otherwise go to 6*

6- *Quit*

$MaxTY_e$  is the final result of the optimization procedure. The associated irrigation levels and  $Land$  used are the required information that maximizes the total expected yield of the corresponding crop over the land of interest. The proposed procedure will be illustrated by the following example.

### **Example 5.3**

The example will consider the output of example 5.1 above with inputs in Table 5.4. The maximum yield in case of applying full water requirement for each growth stage of the crop life is 20,541 Ton/Ha (calculated using water-yield sub-model). This example will consider both situations of whether or not the cropping land is initially constrained.

In the first case (No land constraints), the maximum and minimum land that can be used are calculated according to equations (5.15) and (5.16), respectively for each irrigation season. The minimum value over all maximum lands for each irrigation season is 18.5 hectares. The minimum land that can be used is 9 hectares. The

maximum value of the total expected yield according to the input taken from Tables 5.1 and 5.10 is given in Table 5.11.

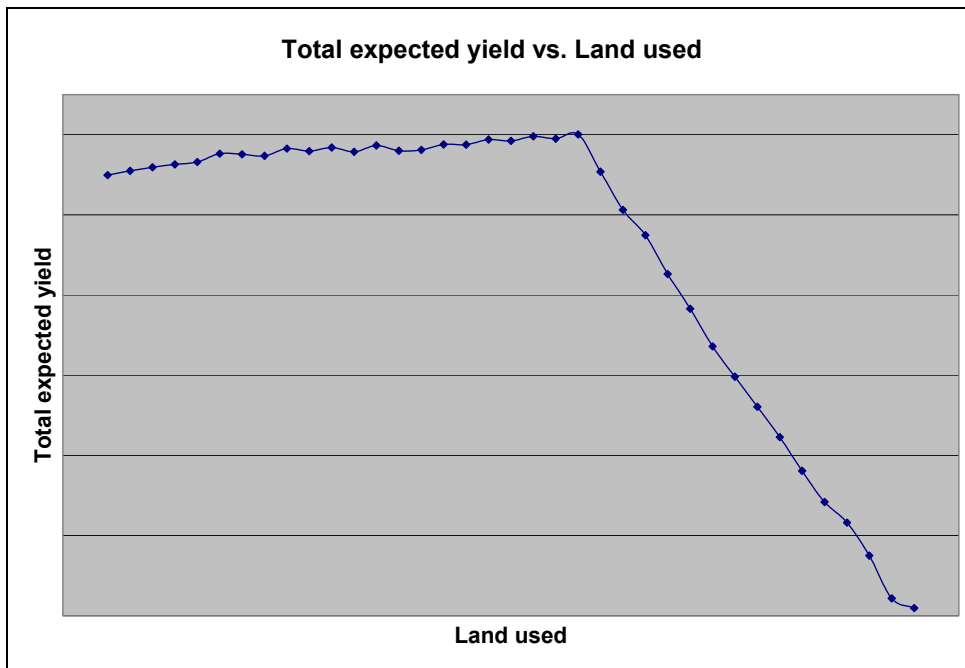
Table 5.11 shows that the maximum total expected yield is obtained when selecting the maximum land associated with the minimum irrigation level (maximum deficit). It also shows that the deficit is applied only in Season 4 (the critical season). While for other seasons, full irrigation levels are selected as the available amount of water exceeds the full requirement for the crop over the selected area. Any additional amount of irrigation water for the non critical irrigation seasons (seasons 1, 2, 3) will not have any effect on the land to be used nor the total expected yield. Any changes in the available amount of water of the critical irrigation season (season 4) however will affect the output of the control sub-model. Different scenarios of available water for irrigation season 4 are proposed and the associated output is given in Tables 5.12 to 5.16.

From the output tables, we find that the area of land to be used increases while increasing the available amount of irrigation water in the critical season, Tables 5.11 to 5.13, until this season (season 4) is no more considered as critical season, Tables 5.14 to 5.16. Therefore, any increment in the available water will not affect the solution.

When the land is constrained, the model will compare the value of the land constraint with the calculated value of the maximum land that can be used and will select the smallest value to be the land constraint. Different values of land constraint will be presented

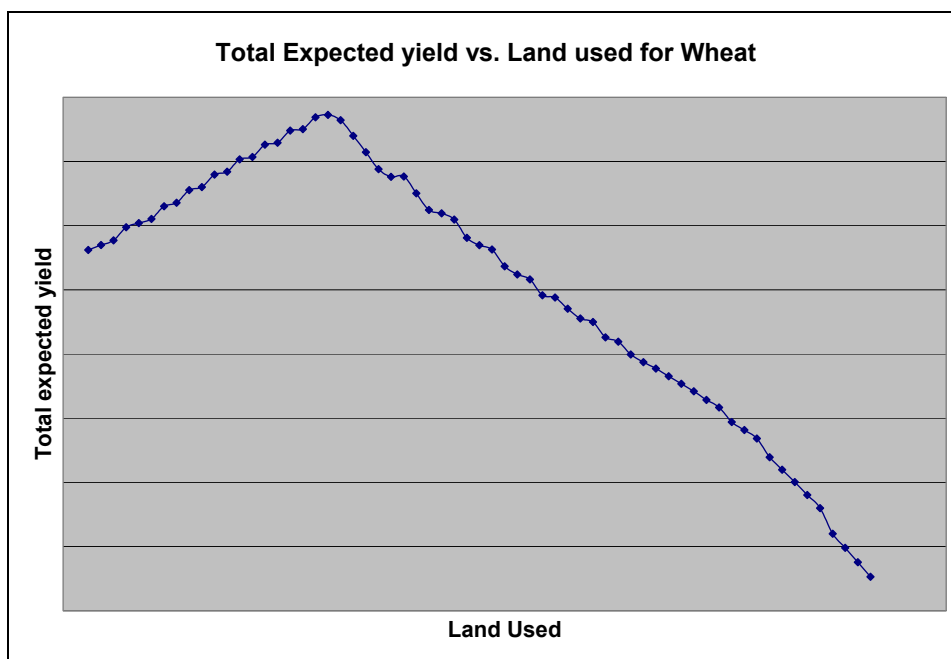
The input information for the example is taken from Table 5.1 for the season water availability and Table 5.10 for Green Maize.

The relation between the total expected yield and the land used is shown in Figure 5.1. This figure shows that the value of the total expected yield increases when increasing the land used (i.e., when increasing the deficit), until it reaches a point after which any more deficit will reduce the total expected yield significantly.

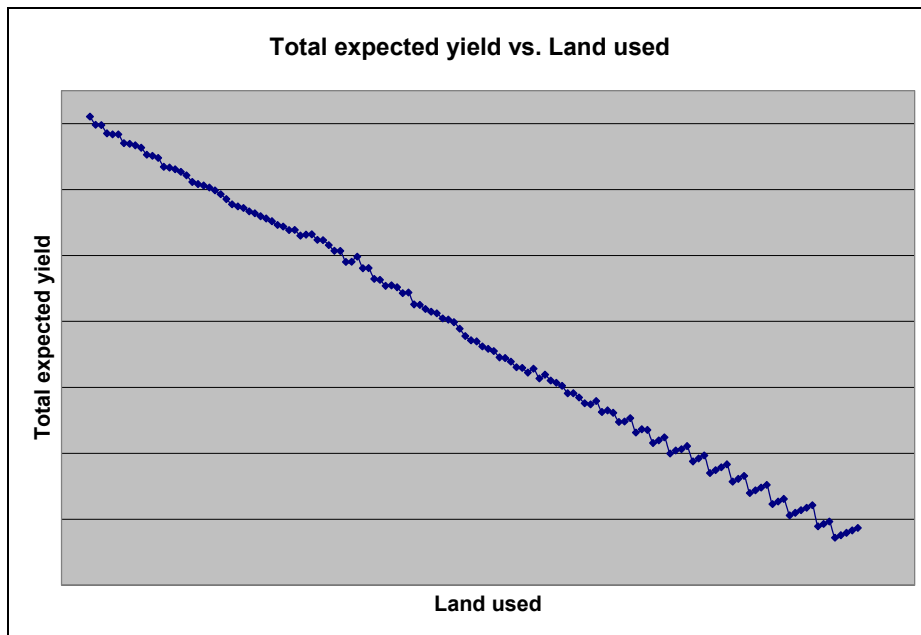


**Figure 5.2: Total Expected yield vs. land used for Green Maize.**

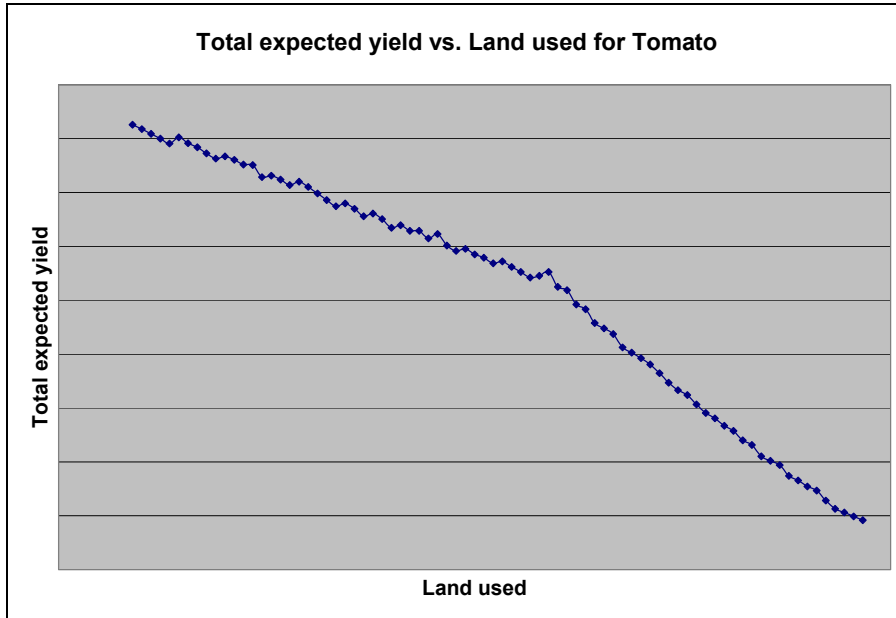
Other behaviors with respect to water deficit can be depicted from the figures below for wheat, onion, and tomato using the single crop model.



**Figure 5.3: Total Expected yield vs land used for Wheat calculated by the Single crop model.**



**Figure 5.4: Total Expected yield vs land used for Onion calculated by the Single crop model.**



**Figure 5.5: Total Expected yield vs land used for Tomato.**

Figure 5.2 shows that wheat has a similar behavior to green maize, with respect to deficit. Figure 5.3 however shows that the maximum total yield for onion is associated with the full irrigation level. In other words, for the case of onion, it is not

beneficial to apply deficit irrigation. Also, Figure 5.4 shows that tomato has a similar behavior to onion with respect to deficit irrigation.

The results above are consistent with those of the study [6] which specify that deficit irrigation is not necessarily favorable with all crops. The single crop model determines in particular to which crops it is adequate to apply deficit irrigation and to what level. The unconstrained version of land also offers the best tradeoff between land use and level of deficit to apply (by considering the optimal combination over the different growth stages).

. Total available water per season (m3)				No Land Constraints												
Season 1	Season 2	Season 3	Season 4	Total expected yield (ton)	Expected yield per hectare (ton/Ha)	Land Used (Ha)	Optimal Irrigation levels									
200,000	300,000	140,000	80,000				Stage 1,1	Stage 1,2	Stage 2	Stage 3,1	Stage 3,2	Stage 4				
Total water consumption (m3)				0	8,192	96.14	78.16	284.822	15.396	18.5	1	1	1	1	0.5	0.5

**Table 5.11: Output for Green Maize calculated by Control sub model according to irrigation season inputs**

Total available water per season (m3)				No Land Constraints												
Season 1	Season 2	Season 3	Season 4	Total expected yield (ton)	Expected yield per hectare (ton/Ha)	Land Used (Ha)	Optimal Irrigation levels									
200,000	300,000	140,000	90,000				Stage 1,1	Stage 1,2	Stage 2	Stage 3,1	Stage 3,2	Stage 4				
Total water consumption (m3)				0	9.299	109.132	88.723	323.311	15.396	21	1	1	1	1	0.5	0.5

**Table 5.12: Output for Green Maize calculated by Control sub model according to irrigation season inputs**

Total available water per season (m3)				No Land Constraints									
Season 1	Season 2	Season 3	Season 4										
200,000	300,000	140,000	110,000	Total expected yield (ton)	Expected yield per hectare (ton/Ha)	Land Used (Ha)	Optimal Irrigation levels						
Total water consumption (m3)							Stage 1,1	Stage 1,2	Stage 2	Stage 3,1	Stage 3,2	Stage 4	
0	11513	135112	109000	400.29	15.396	26	1	1	1	1	0.5	0.5	

**Table 5.13: Output for Green Maize calculated by Control sub model according to irrigation season inputs**

Total available water per season (m3)				No Land Constraints									
Season 1	Season 2	Season 3	Season 4										
200,000	300,000	140,000	140,000	Total expected yield (ton)	Expected yield per hectare (ton/Ha)	Land Used (Ha)	Optimal Irrigation levels						
Total water consumption (m3)							Stage 1,1	Stage 1,2	Stage 2	Stage 3,1	Stage 3,2	Stage 4	
0	14613	138591	139422	461.87	14	33	1	1	1	0.5	0.5	0.5	

**Table 5.14: Output for Green Maize calculated by Control sub model according to irrigation season inputs**

Total available water per season (m3)				No Land Constraints									
Season 1	Season 2	Season 3	Season 4										
200,000	300,000	140,000	160,000	Total expected yield (ton)	Expected yield per hectare (ton/Ha)	Land Used (Ha)	Optimal Irrigation levels						
Total water consumption (m3)							Stage 1,1	Stage 1,2	Stage 2	Stage 3,1	Stage 3,2	Stage 4	
0	14392	139731	159992	491.265	15.116	32.5	1	1	1	0.55	0.65	0.5	

**Table 5.15: Output for Green Maize calculated by Control sub model according to irrigation season inputs**

Total available water per season (m3)				No Land Constraints									
Season 1	Season 2	Season 3	Season 4										
200,000	300,000	140,000	220,000	Total expected yield (ton)	Expected yield per hectare (ton/Ha)	Land Used (Ha)	Optimal Irrigation levels						
Total water consumption (m3)							Stage 1,1	Stage 1,2	Stage 2	Stage 3,1	Stage 3,2	Stage 4	
0	14392	139731	219088	571.845	17.595	32.5	1	1	1	0.55	1	0.55	

**Table 5.16: Output for Green Maize calculated by Control sub model according to irrigation season inputs**

Total available water per season (m3)				Land Constraint = 20Ha								
Season 1	Season 2	Season 3	Season 4	Total expected yield (ton)	Expected yield per hectare (ton/Ha)	Land Used (Ha)	Optimal Irrigation levels					
200,000	300,000	140,000	80,000				Stage 1,1	Stage 1,2	Stage 2	Stage 3,1	Stage 3,2	Stage 4
Total water consumption (m3)												
0	8192	96140	78160	284.822	15.396	18.5	1	1	1	1	0.5	0.5

**Table 5.17: Output for Green Maize calculated by Control sub model according to irrigation season inputs**

Total available water per season (m3)				Land Constraint = 17 Ha								
Season 1	Season 2	Season 3	Season 4	Total expected yield (ton)	Expected yield per hectare (ton/Ha)	Land Used (Ha)	Optimal Irrigation levels					
200,000	300,000	140,000	80,000				Stage 1,1	Stage 1,2	Stage 2	Stage 3,1	Stage 3,2	Stage 4
Total water consumption (m3)												
0	7528	88344	79732	272.832	16.049	17	1	1	1	1	0.6	0.5

**Table 5.18: Output for Green Maize calculated by Control sub model according to irrigation season inputs**

Total available water per season (m3)				Land Constraint = 15 Ha								
Season 1	Season 2	Season 3	Season 4	Total expected yield (ton)	Expected yield per hectare (ton/Ha)	Land Used (Ha)	Optimal Irrigation levels					
200,000	300,000	140,000	80,000				Stage 1,1	Stage 1,2	Stage 2	Stage 3,1	Stage 3,2	Stage 4
Total water consumption (m3)				250.531	16.702	15	1	1	1	1	0.7	0.5
0	6642	77951	77332									

**Table 5.19: Output for Green Maize calculated by Control sub model according to irrigation season inputs**

Total available water per season (m3)				Land Constraint = 13 Ha								
Season 1	Season 2	Season 3	Season 4	Total expected yield (ton)	Expected yield per hectare (ton/Ha)	Land Used (Ha)	Optimal Irrigation levels					
200,000	300,000	140,000	80,000				Stage 1,1	Stage 1,2	Stage 2	Stage 3,1	Stage 3,2	Stage 4
Total water consumption (m3)				234.109	18.01	13	1	1	1	1	0.9	0.5
0	5757	67558	79118									

**Table 5.20: Output for Green Maize calculated by Control sub model according to irrigation season inputs**

Total available water per season (m3)				Land Constraint = 11 Ha								
Season 1	Season 2	Season 3	Season 4	Total expected yield (ton)	Expected yield per hectare (ton/Ha)	Land Used (Ha)	Optimal Irrigation levels					
200,000	300,000	140,000	80,000				Stage 1,1	Stage 1,2	Stage 2	Stage 3,1	Stage 3,2	Stage 4
Total water consumption (m3)												
0	4871	57164	78330	212.119	19.28	11	1	1	1	1	1	0.65

**Table 5.21: Output for Green Maize calculated by Control sub model according to irrigation season inputs**

Total available water per season (m3)				Land Constraint = 9 Ha								
Season 1	Season 2	Season 3	Season 4	Total expected yield (ton)	Expected yield per hectare (ton/Ha)	Land Used (Ha)	Optimal Irrigation levels					
200,000	300,000	140,000	80,000				Stage 1,1	Stage 1,2	Stage 2	Stage 3,1	Stage 3,2	Stage 4
Total water consumption (m3)												
0	3985	46771	76048	186.615	20.735	9	1	1	1	1	1	1

**Table 5.22: Output for Green Maize calculated by Control sub model according to irrigation season inputs**

Total available water per season (m3)				Land Constraint = 7 Ha								
Season 1	Season 2	Season 3	Season 4	Total expected yield (ton)	Expected yield per hectare (ton/Ha)	Land Used (Ha)	Optimal Irrigation levels					
Total water consumption (m3)							Stage 1,1	Stage 1,2	Stage 2	Stage 3,1	Stage 3,2	Stage 4
200,000	300,000	140,000	80,000	145.145	20.735	7	1	1	1	1	1	1
0	3100	36377	59149									

**Table 5.23: Output for Green Maize calculated by Control sub model according to irrigation season inputs**

### **5.7 Interfacing with the Multi-crop Model**

As mentioned earlier, the single crop model also prepares a set of combinations of irrigation levels of the crop growth stages in order to be used by the multi-crop model (to be discussed in the next chapter). The combinations are determined through interfacing with the dynamic programming model.

The control model determines the possible irrigation levels and the associated water consumption for each growth stage of the crop life. This is associated with minimum and maximum water requirement of the crop through the whole growing duration. The difference between the minimum and maximum water requirement is discretized into a number of levels, taken to be 20 levels. This number of levels is adjustable without affecting the model formulation. Also, this number is meant to be more than the number of possible irrigation levels at each growth stage. Furthermore, it gives a reasonable variety of different water consumptions to the multi-crop model that may be consistent with different scenarios of water availability. Increasing the number of levels will increase the size of the multi-crop model and therefore the computation time, but will also increase the accuracy of the multi-crop model's output.

For each level of water starting from the minimum to the maximum requirement, the control sub-model interfaces with the dynamic programming model to determine the optimal allocation of the available water through the different growth stages. The result provides the most promising 20 combinations of irrigation levels, water consumption, and the resulting expected yield for each combination. An example is given in appendix (B)

## Chapter 6

---

# The Multi-Crop Model

### 6.1. Overview of the model

The model considers multiple crops competing for both land and irrigation water. The question the model attempts to solve would be: what is the best allocation of land and water, both in time and space, among crops in order to maximize the net profit of a given farmer/agricultural company. Water allocation, in this case, should be directed towards increasing the yield of the most profitable crops. Therefore, low-value crops will have comparatively low shares of water and land and high-value crops will have the high shares.

This model tries to determine the optimal cropping pattern of the land of interest for a given year. Each selected crop must be cropped according to its cropping period. Therefore, some crops may not be following others in the cropping plan because of overlaps in their cropping periods. Moreover, some crops are considered as good successors/predecessors of other crops if not only it is feasible to crop them consecutively (in the right order)" wrong" predecessor.

Maximizing profit depends in part on the market conditions. For some farmers, some crops can not be totally marketed if the production exceeds some particular level. This suggests setting in the model an upper bound for the maximum quantity to be produced. Similarly, some minimum amounts of some crops may be dictated by contract commitment with retail companies. This may be the case even if those crops where not considered as high profit crops.

The model will account for the market requirements while searching for the optimal policy. The model will also consider the seasonality in water availability. It is assumed

that the year consists of four different irrigation seasons, each with a certain pumping capacity. Total water allocation for all selected crops in a given season can not exceed the pumping capacity of each irrigation season. Moreover, the model will account for the irrigation efficiency of irrigation systems used in different pieces of land. In addition, the model will consider the fact that part of the land might be still occupied by some crops at the start of executing the cropping plan but would be made available later and could be used by some successors.

## **6.2. Objective of the model**

The model objective is to maximize the expected profit of the land of interest by determining the following:

- The area of land to be allocated to each selected crop
- The best sequence of the crops to be grown over the same land accounting for crop rotation
- The irrigation level to be applied to each growth stage of the crop life of selected crops
- The cumulative amount of water to be consumed by each selected crop
- The expected yield of each selected crop according to the selected cropping area and the selected combination of irrigation levels
- The expected profit of each selected crop

## **6.3. Methodology**

An optimization model is developed using various inputs (as explained below). The model uses several sets of constraints, namely water and land availability constraints, market constraints, time constraints and mass balance constraints. Crop rotation is considered by introducing a penalty factor named “predecessor coefficient”. The values of this factor vary between 0 and 1. This factor will multiply the expected perfect yield of the successor. In particular, the low values of the predecessor coefficient indicate that the successor will suffer a great yield reduction if cropped after the given predecessor.

In addition, a binary variable is introduced in the model to control the time feasibility of selecting two crops to be grown in a row. This variable is of great importance in the optimization procedure as it significantly reduces the size of the problem by eliminating the infeasible combinations of crops. The model receives the information regarding each candidate crop from the single crop model. Other information is also received as inputs (as explained later). The model is implemented using LNIGO 8.0 software to determine the optimal policy.

#### **6.4. Inputs**

The inputs of the model can be categorized in three different categories:

1- Land inputs which cover:

- Current crops that are previously grown over the land of interest

Also, for each of the current crops, the inputs cover:

- Cropped area
- Current crop ending date (including the harvesting duration)
- Irrigation system efficiency

2- Irrigation Season inputs

- Available amount of water for each irrigation season

3- Candidate Crops inputs:

- Earliest cropping date
- Latest Cropping date
- Growth period in days
- Harvesting Period in days
- Selling Price per Ton
- Cost Per Hectare
- Market constraints (minimum & maximum requirements)
- The most promising combinations of the different irrigation levels for each growth stage of the crop life (say the best 20 combinations of levels. These are taken from the single crop model discussed in the

previous chapter. Moreover, the number of promising levels is adjustable.)

- Water consumption in each season associated with each combination of the irrigation levels for the crop growth stages (could be taken from the single-crop model)
- Expected yield associated with each combination (could be taken from the single crop model)
- Predecessor coefficients (set arbitrarily but reflecting the adequacy of crop successions)
- Binary variable called *Possible* that takes the value one if the predecessor coefficient is positive (input value to the model)

## **6.5. Outputs**

The outputs are categorized in two categories:

### **6.5.1. General output**

- Total water to be consumed in each irrigation season
- Total expected yield
- Total expected profit

### **6.5.2. Detailed output**

For each selected crop, the model will determine:

- Area Allocated
- Immediate predecessor
- Specific piece of land (identified by the existing crops at the start of the cropping plan)
- Cropping date
- End of growth date
- End of harvesting date

- Water to be consumed
- Selected irrigation level for each growth period

## 6.6. Notations

Before going in details in the objective function and constraints, we will introduce the notations that will be used in both the mathematical model and LINGO code.

### 6.6.1. Land notations

$m$ : Land index referring to the corresponding existing crop,  $m = 1, 2, \dots, M$ , where  $M$  is the total number of available pieces of land and therefore of existing crops right before the cropping plan of interest.

$L\_C_m$ : The name of the Crop that currently exists over land  $m$ , possibly none

$L\_A_m$ : area of land  $m$

$L\_IrrSysEff_m$ : Irrigation system efficiency in land  $m$

$L\_CCED_m$ : Current crop ending date (ready for harvesting)

### 6.6.2. Irrigation Season Notations

$s$ : The index of seasons,  $s = 1, 2, 3, 4$  as we assume that the year consists of 4 seasons.

$S\_N_s$ : Name of season  $s$

$S\_AW_s$ : Available water in season  $s$  (i.e., total groundwater that could be pumped over the season)

$S\_WC_s$ : Total water to be consumed in season  $s$  (decision variable).

### 6.6.2. Crop Notations

$i$ : index of any candidate crop,  $i = 1, 2, \dots, I$ , where  $I$  is the total number of candidate crops. (The model will consider a given crop as a candidate crop if the maximum production requirement is taken to be greater than zero.)

$k$ : index of the predecessor of a given crop  $i$ .  $k = 1, 2, \dots, K$ , where  $K$  is the total number of possible predecessors of crop  $i$  that can be either a candidate or existing crop. That is,  $K = M \cup I$ . (A crop is identified to be a possible predecessor of another crop if the corresponding predecessor coefficient is not zero. This coefficient would be zero if the two crops are naturally incompatible.)

$C\_SeP_i$ : Selling price of crop  $i$

$C\_Ct_i$ : Cost of crop  $i$

$C\_SDmn_i$ : Earliest start of growing crop  $i$

$C\_SDmx_i$ : Latest start of growing crop  $i$

$C\_GP_i$ : Growth period of crop  $i$

$C\_MxR_i$ : Maximum requirement of crop  $i$  (market constraint)

$C\_MnR_i$ : Minimum requirement of crop  $i$  (market constraint)

$C\_NIL_i$ : Number of combinations of irrigation levels for crop  $i$  over the different growth stages

$C\_PTR_i$ : Harvesting duration of crop  $i$

$CL\_WCSs_{ij}$ : Water consumed in irrigation season  $s$  by crop  $i$ , using the  $j^{\text{th}}$  combination of irrigation levels,  $s = 1, \dots, 4$

$CL\_PEY_{ij}$ : Perfect expected yield of crop  $i$  under the  $j^{\text{th}}$  combination of irrigation levels (when cropped after a perfect predecessor)

$CP\_PC_{ik}$ : Predecessor coefficient of crop  $i$  if cropped after crop  $k$

$CP\_Possible_{ik}$ : Discussed earlier,

$CP\_Possible_{ik} = 1$  if the predecessor coefficient  $> 0$   
and zero otherwise

$CL\_FD_{ikm}$ : End of growth period of crop  $i$  having predecessor  $k$  over land  $m$

$CL\_ED_{ikm}$ : Date when the land is ready for the next exploitation after  
harvesting crop  $i$  with predecessor crop  $k$  over land  $m$ , that is

$$CL\_ED_{ikm} = CL\_FD_{ikm} + C\_PTR_i$$

$CL\_CCED_{km}$ : End of harvest period of crop  $k$  over land  $m$

$CL\_EY_{ikm}$ : Expected yield of crop  $i$  when grown after crop  $k$  over land  $m$   
(that is,  $CL\_EY_{ikm} = CL\_PEY_{ij} * CP\_PC_{ik}$ )

$LC\_A_{km}$ : Area allocated to crop  $k$  over land  $m$

$CL\_Area_{ikm}$ : Area allocated to crop  $i$  over crop  $k$  over land  $m$

Additional notation will be introduced when needed.

### 6.6.3. Decision variables

$Area_{ijkm}$ : Area of land  $m$  allocated to crop  $i$  with the  $j^{\text{th}}$  combination of irrigation  
level after predecessor  $k$

$CL\_CD_{ikm}$ : Cropping date of crop  $i$  after crop  $k$  over land  $m$

$Reject_{ikm}$ : A binary variable indicating whether crop  $i$  can be grown over crop  $k$   
over land  $m$  from time-feasibility point of view

$Reject_{ikm} = 1$ , if it can not be grown

$Reject_{ikm} = 0$  otherwise

$Select_{ikm}$ : A binary variable that takes the value one if crop  $i$  is selected over crop  $k$  over land  $m$ , and zero otherwise. This decision variable activates the constraints that determine the cropping date of crop  $i$ .

**Note:** The decision variable  $Select_{ikm}$  is not required, as  $Reject_{iklm}$  is enough to do the job. While adding this decision variable increases the size of the problem, it reduces the computational space, leading to shorter computer run. This is justified from some computational instances.

### 6.7. Objective Function:

The objective is to maximizing the total expected profit of the entire land over the planning horizon.

In general, the profit resulting from each crop can be calculated as follows:

$$Profit = production \times selling\ price\ (per\ ton) - land\ allocated \times cost\ (per\ he) \quad (6.1)$$

The production of a given crop can be calculated as:

$$Production = area\ of\ land\ allocated \times expected\ yield\ per\ he \quad (6.2)$$

To incorporate the effect of the predecessor on the yield, we introduce the predecessor coefficient and we multiply the perfect yield (obtained by having a perfect predecessor) by the predecessor coefficient to obtain:

$$Expected\ yield = Predecessor\ coefficient \times Expected\ Perfect\ yield \quad (6.3)$$

If crop  $i$  is selected to be grown on more than one piece of land and if on each piece we have a different combination of irrigation levels and a different predecessor, then the total production of crop  $i$  will have the following equation:

$$C\_TEY_i = \sum_{k=1}^K \left[ CP\_PC_{ik} \times \sum_{m=1}^M \sum_{j=1}^J Area_{ijkm} \times CL\_EY_{ij} \right] \quad (6.4)$$

The profit equation of crop  $i$  is given by

$$C\_TEP_i = \left[ \begin{array}{l} \sum_{k=1}^K \left[ CP\_PC_{ik} \times \sum_{m=1}^M \sum_{j=1}^J Area_{ijklm} \times CL\_EY_{ij} \right] \times C\_Sep_i \\ - \sum_{m=1}^M \sum_{k=1}^K \sum_{j=1}^J Area_{ijkm} \times C\_Ct_i \end{array} \right] \quad (6.5)$$

Taking the area of land from both sides, the equation will be of the form:

$$C\_TEP_i = \sum_{m=1}^M \sum_{k=1}^K \sum_{j=1}^J Area_{ijkm} \times [CP\_PC_{ik} \times CL\_EY_{ij} \times C\_Sep_i - C\_Ct_i] \quad (6.6)$$

$$Total\ Net\ Profit = \sum_{i=1}^I C\_TEP_i \quad (6.7)$$

That is,

$$Total\ Net\ Profit = \sum_{i=1}^I \sum_{m=1}^M \sum_{k=1}^K \sum_{j=1}^J Area_{ijkm} \times [CP\_PC_{ik} \times CL\_EY_{ij} \times C\_Sep_i - C\_Ct_i] \quad (6.8)$$

Equation (6.8) is the objective function we are interested in maximizing.

## 6.8. Constraints

The constraints are categorized in five classes as follows

### 6.8.1. Market constraints

The total yield of each crop should fall between the maximum and minimum market requirements. That is, for each crop, we must have

$$\sum Production \leq Maximum\ market\ constraints$$

$$\sum Production \geq Minimum\ market\ constraints$$

This is formulated as follows: for each crop  $i$

$$\sum_{m=1}^M \sum_{k=1}^K \sum_{j=1}^J Area_{ijkm} \times CP_{ik} \times CL_{EY_{ij}} \leq C_{MxR_i} \quad (6.12)$$

$$\sum_{m=1}^M \sum_{k=1}^K \sum_{j=1}^J Area_{ijkm} \times CP_{ik} \times CL_{EY_{ij}} \geq C_{MnR_i} \quad (6.13)$$

### 6.8.2. Land constraints

The sum of areas allocated to all crops that follow a certain predecessor must not exceed the area of land allocated to this predecessor in the previous exploitation.

For each Predecessor  $k$  and for each Land  $m$ :

$$\sum_{i=1}^I \sum_{j=1}^J Area_{ijkm} \leq LC_{A_{km}} \quad (6.14)$$

The value of  $LC_{A_{km}}$  is calculated by summing over all combinations of irrigation levels and predecessors, using the following equation:

$$LC_{A_{im}} = \sum_{j=1}^J \sum_{k=1}^K Area_{ijkm} \quad (6.15)$$

Equation (6.14) will change to the following form when taking the decision on the crops that will immediately take over those of last exploitation (i.e., when  $k = m$ ):

$$\sum_{i=1}^I \sum_{j=1}^J Area_{ijmm} \leq LC_{A_{mm}} \quad (6.16)$$

That is, the total area to be allocated to crop  $i$  over all combinations of irrigation levels over land  $m$ , can not exceed the area of land  $m$ ,  $LC_{A_{mm}}$

### 6.8.3. Water Constraints

Water to be consumed during each irrigation season can be calculated as follows:

$$S_{WC_s} = \sum_{m=1}^M \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K [WC_{ij} \times Area_{ijkm}] / IrrSysEff_m \quad (6.17)$$

The total amount of water to be consumed in each irrigation season can not exceed seasonal water availability.

$$S\_WC_s \leq S\_WA_s \quad (6.18)$$

#### 6.8.4. Time constraints

As mentioned earlier, each crop has an earliest cropping date and a latest cropping date and both dates can not be violated. Consequently, some crops can not follow others in the cropping pattern, because of cropping duration overlap. This situation is considered in the optimization model by introducing a binary variable named  $Possible_{ik}$  the value of which will take one if no time violation will occur and zero otherwise. This value will be calculated outside the optimization model and will be inserted as an input to the model. The value of  $Possible_{ik}$  will determine a preliminary set of possible successors for each crop. Moreover, in some cases a crop (say crop  $i$ ) may be considered a feasible successor of another crop (say crop  $k$ ), but the decision to be made on the cropping date of the first crop (crop  $k$ ) may affect the feasibility of being followed by the other crop (crop  $i$ ) as the finishing date of  $k$  may exceed the latest start of  $i$ . An illustration is given in Appendix D.

Such situations are approached in the model by introducing a binary decision variable named  $Reject_{ikm}$ . This variable indicates whether crop  $i$  can follow crop  $k$  over land  $m$ , accounting for the cropping date decisions, as follows:

$$Reject_{ikm} = 0 \text{ if crop } i \text{ can follow crop } k \text{ over land } m$$

$$Reject_{ikm} = 1 \text{ otherwise}$$

The value of  $Reject_{ikm}$  is related to the input value of  $Possible_{ik}$  as follows:

$$\text{If } Possible_{ik} = 0, \text{ then } Reject_{ikm} = 1.$$

This is the special case where crop  $i$  can never be cropped after  $k$ , independently of the cropping date decisions (always an overlap will occur). The previous relation can be expressed mathematically using the following constraints:

For each crop  $i$ , for each crop  $k$ , and for each piece of land  $m$ :

$$Reject_{ikm} \geq 1 - Possible_{ik} \quad (6.20)$$

If  $Possible_{ik} = 1$ , then the value of  $Reject_{ikm}$  will be determined from the optimization model according to the finishing date of crop  $k$  and the latest start of crop  $i$  as follows:

$$Reject_{ikm} = 0 \text{ if the latest cropping date of crop } i \text{ is greater than the finishing date of } k \text{ (including the harvesting period)}$$

$$Reject_{ikm} = 1 \text{ otherwise}$$

The above relation can be expressed mathematically as follows:

At the start of the planning horizon, (only crop  $m$  is grown) the value of  $Reject_{imm}$  will be determined by the following constraint:

$$CL\_CCED_{imm} + 1 - C\_SDmx_i - L \times Reject_{imm} \leq 0 \quad (6.21)$$

where,  $L$  is a very large number.

If however, crop  $i$  is considered after some crops, where the last one is  $k$  and the predecessor of  $k$  is  $l$  (possibly the initial crop  $m$ ), then (6.21) will change to (6.22), which is given for any candidate crop  $i$  over a current crop  $k$  over crop  $l$  over land  $m$ , by

$$CL\_ED_{klm} + 1 - C\_SDmx_i - L \times Reject_{ikm} \leq 0 \quad (6.22)$$

The structure of (6.22) is similar to that of (6.21) except for the input value  $CL\_CCED_{imm}$  in (6.21) which is replaced by the decision variable  $CL\_ED_{klm}$  in (6.22), where the first term expresses the ending date of initial crop  $m$ , while the other term ( $CL\_ED_{klm}$ ) expresses the ending date of crop  $k$  grown over crop  $l$  over land  $m$ .

Note that, if the value of  $Possible_{ik}=0$  in (6.20), then  $Reject_{ikm}=1$  and (6.21) would always apply since  $L$  is very large. Otherwise, a comparison between the latest cropping date of crop  $i$  and the ending time of predecessor  $k$  will be made in setting the value of  $Reject_{ikm}$ .

The other set of constraints related to time constraints consists of those concerned with selecting the cropping date of the candidate crop.

The cropping date of any candidate crop is considered as a decision variable that must be determined in the range of earliest to latest cropping date. That is, we want to have a constraint of the following form:

for any candidate crop  $i$ , over crop  $k$ , over land  $m$ :

$$C\_SDmn_i \leq CL\_CD_{ikm} \leq C\_SDmx_i \quad (6.23)$$

However, (6.23) is independent of the decision on the area to be allocated to crop  $i$ . For the model to satisfy constraints (6.23) in both cases, whether crop  $i$  is selected or not, we introduce the following binary decision variable

$Select_{ikm}=1$  if crop  $i$  is selected to be cropped after crop  $k$  over land  $m$ .

$Select_{ikm}=0$  otherwise.

The procedure for calculating the value of this variable will be discussed in the next section (Mass balance constraints). After introducing  $Select_{ikm}$ , constraints (6.23) will be replaced by the following two sets of constraints:

For any crop  $i$  for each crop  $k$  and for each land  $m$ :

$$CL\_CD_{ikm} - C\_SDmx_i \times Select_{ikm} \leq 0 \quad (6.24)$$

$$CL\_CD_{ikm} - C\_SDmn_i \times Select_{ikm} \geq 0 \quad (6.25)$$

Moreover, at the start of the planning horizon, (only crop  $m$  is grown) cropping date of crop  $i$  must exceed the ending date of its predecessor  $m$ ,

$$CL\_CD_{imm} - CL\_CCED_{imm} - 1 \geq -L \times (1 - Select_{imm}) \quad (6.26)$$

If however, crop  $i$  is considered after some crops, where the last one is  $k$  and the predecessor of  $k$  is  $l$  (possibly the initial crop  $m$ ), then (6.26) will change to (6.27), which is given for any candidate crop  $i$  over a current crop  $k$  over crop  $l$  over land  $m$ , by

$$CL\_CD_{ikm} - CL\_ED_{klm} - 1 \geq L \times (Select_{ikm} - 1) \quad (6.27)$$

Ending date of any selected crop should not exceed the planning horizon (denoted by  $PIH$ ), which is assumed in the implementation to be one year (365 days). This is purely for practical considerations, so that we include its corresponding profit within the planning horizon.

For any crop  $i$  over crop  $k$  over land  $m$ :

$$CL\_ED_{ikm} \leq PIH \quad (6.28)$$

#### 6.8.5. Mass balance constraints

In the previous discussion, we introduced the decision variable  $Reject_{ikm}$  that indicates whether the candidate crop  $i$  can be considered as a feasible successor of crop  $k$  or not. The model checks the value  $Reject_{ikm}$  before allocating any area to the candidate crop  $i$  as follows:

For each crop  $i$  over crop  $k$  over land  $m$ :

$$\sum_{j=1}^J Area_{ijkm} \leq L \times (1 - Reject_{ikm}) \quad (6.29)$$

This set of constraints means that if  $Reject_{ikm} = 1$ , then the right term will be equal to zero and the summation of all the area, with different irrigation levels, allocated to the crop  $i$  (over crop  $k$  over land  $m$ ) is equal to zero.

If the model allocates any positive value to  $Area_{ijkm}$  then the decision variable  $select_{ikm}$  must take the value 1. Also, if  $select_{ikm}$  takes the value zero, then the allocated area to crop  $i$  must be zero. This suggests adding the following sets of constraints:

For each crop  $i$  over crop  $k$  over land  $m$ :

$$\sum_{j=1}^J Area_{ijkm} \leq L \times Select_{ikm} \quad (6.30)$$

$$\sum_{j=1}^J Area_{ijkm} \geq (L)^{-1} \times Select_{ikm} \quad (6.31)$$

The ending date of any candidate crop is the summation of the cropping date and the harvesting period. This is expressed as follows:

$$CL\_ED_{ikm} = CL\_CD_{ikm} + (C\_GP_i + C\_PTR_i) \times Select_{ikm} \quad (6.32)$$

$$Reject_{ikm} = 0 \text{ or } 1. \quad (6.33)$$

$$Select_{ikm} = 0 \text{ or } 1. \quad (6.34)$$

$$\textit{The rest of decision variables are non-negative} \quad (6.35)$$

## 6.9. Illustration

We consider a farm of 180 ha, where initially 60 ha were cropped with alfalfa, 60 ha with potato and the remaining 60 ha were empty. The cropping plan is for the year starting at October 1<sup>st</sup> 2004. Land information is given in Table 6.1 below. This information is based mainly on [6]. Crop information is given in Table 6.2 below. The data is a mix of published data [6] and field data [1]. Also, the water-season information is given in Table 6.3 below and it is based on hypothetical data. In addition, values for predecessor coefficients are given in Table 6.4 below. These coefficients are based on hypothetical values. The set of the most promising combinations of irrigation levels for each candidate crop are obtained using the single-crop model and they are given in Appendix B.

Land	L_A	End of Growth	Harvesting period (Days)	L_CCED	L_IrrSystem	L_IrrSysEff
Empty	60	01/01/2004	0	1	Pivot system_2	0.95
Alfalfa	60	10/20/2004	15	294	Pivot system_2	0.93
Potato	60	11/15/2004	17	320	Pivot system_2	0.96

**Table 6.1: Land Information**

Crop Name	Earliest Cropping date	Latest Cropping date	Expected Selling Price (SR/Ton)	Expected Cost (SR/hHa)	Growth Period (Days)	Harvesting Period (Days)	Requirement Max (Ton)	Requirement Min (Ton)	Possible Irrigation Levels
Grain maize	426	74	1500	14000	140	15	200	70	20
Onion	288	305	1700	20000	100	12	150	100	20
Tomato_1	15	46	2000	1000	100	20	100	20	20
Wheat	349	365	780	5000	120	20	400	50	20

**Table 6.2: Crop Information.**

Season	Starting Date	Total Available water (m3)
1	01-Oct	13000
2	01-Dec	50000
3	15-Mar	30000
4	15-May	30000

**Table 6.3: Water-Season Information.**

Candidate	Predecessor	CP_PC	Possible
Grain maize	Grain maize	0.2	1
Grain maize	Wheat	0.8	1
Grain maize	Onion	0.7	1
Grain maize	Tomato_1	0	0
Grain maize	Empty	0.9	1
Grain maize	Alfalfa	1	1
Grain maize	Potato	0.7	1
Wheat	Grain maize	0.9	1
Wheat	Wheat	0.7	1
Wheat	Onion	1	1
Wheat	Tomato_1	0.8	1
Wheat	Empty	0.7	1
Wheat	Alfalfa	0.6	1
Wheat	Potato	0.5	1
Onion	Grain maize	0.8	1
Onion	Wheat	0.9	1
Onion	Onion	0.5	1
Onion	Tomato_1	0.9	1
Onion	Empty	1	1
Onion	Alfalfa	0.8	1
Onion	Potato	0.6	1
Tomato_1	Grain maize	1	1
Tomato_1	Wheat	0	0
Tomato_1	Onion	0.9	1
Tomato_1	Tomato_1	0.8	1
Tomato_1	Empty	0.9	1
Tomato_1	Alfalfa	0.9	1
Tomato_1	Potato	0.8	1

**Table 6.4: Crop predecessor coefficient Information.**

The optimal solution is given in Tables 6.5-6.7 below.

Season	Starting Date	Total Available water (m3)	Water to be consumed (m3)
1	01-Oct	13000	13000
2	01-Dec	50000	50000
3	15-Mar	30000	30000
4	15-May	30000	19323

**Table 6.5: Seasonal water consumption of optimal exploitation.**

Candidates	Total Area Cropped (Ha)	Total Water Consumed (m3)	Total Expected Yield (Ton)	Total Expected Profit (SR)
GRAIN_MAIZE	3.8	32221	70	57734
ONION	5.1	12999	121	103405
TOMATO_1	8.3	32794	96	184562
WHEAT	10.5	34307	139	55797

**Table 6.6: General output of the optimal exploitation.**

### 6.9.1 Discussion of the results

A very limited area of land is used. This is due to the water availability constraint. When relaxing this constraint, more land is used as shown by the sensitivity analysis (discussed below). Land exploitation is made such that wheat follows tomato and green maize both cropped over the initially empty area. Similarly, onion is cropped over the empty area. The crop predecessor coefficient was critical in assigning the specific location for each crop, as the land having the highest possible predecessor coefficient was always selected. The only cases where the absolutely best predecessor land was not selected was due to crop life conflicts (time constraints). Green maize (for which water requirement was by far the largest) was given its minimum requirement (this was also the case of the various scenarios considered in the sensitivity analysis below). Although, water availability was highly constrained in this example, green maize has used full irrigation levels. This is also partly the case for tomato which was assigned two different combinations of irrigation levels, the full irrigation level and in another area a very close combination from the full water requirement. Wheat was

allocated the irrigation level that realizes the best tradeoff between land expansion and water use, as determined in the single-crop model (see Figure 5.2 of the previous chapter).

### **6.9.2 Sensitivity Analysis**

We performed a sensitivity analysis with respect to four factors; namely water availability, crop predecessor coefficient, minimum market requirements, and profit. The last factor was varied by fixing the selling price and changing the cost. The results are given in Appendix E. They are intuitive and reflect the importance of the different factors. Higher water availability increased the cropped area and also decreased irrigation deficit. The crop predecessor coefficient was critical in determining the sequence of crops to be grown in the same land. This shows that, for an optimal exploitation, it is important to determine not only the adequate crops to be selected and the corresponding area to be used, but also the specific location of each crop in the right sequence. When relaxing the minimum market constraints, green maize share of land was further reduced leading a total yield below the initial minimum requirement. In terms of profit, green maize was comparable to the other crops, but its large water consumption was behind such an allocation. When the profit varied (by changing costs), then the total land exploitation has significantly changed.

Candidates	Immediate Predecessor	Land	Area to be cropped (Ha)	Cropping Date	Harvesting date	Ending Date	Water Consumed (m3)	Irrigation Efficiency	Expected yield (Ton)	Expected Profit (SR)	Irrigation Level
GRAIN_MAIZE	ALFALFA	ALFALFA	3.4	440	566	581	32221	0.93	70	57734	20
ONION	EMPTY	EMPTY	5.1	299	399	411	12999	0.95	121	103405	20
TOMATO_1	EMPTY	EMPTY	8.3	412	512	532	32794	0.95	96	184563	17
WHEAT	EMPTY	EMPTY	10.5	350	470	490	34307	0.95	139	55797	12

**Table 6.7: Detailed output of the optimal exploitation.**

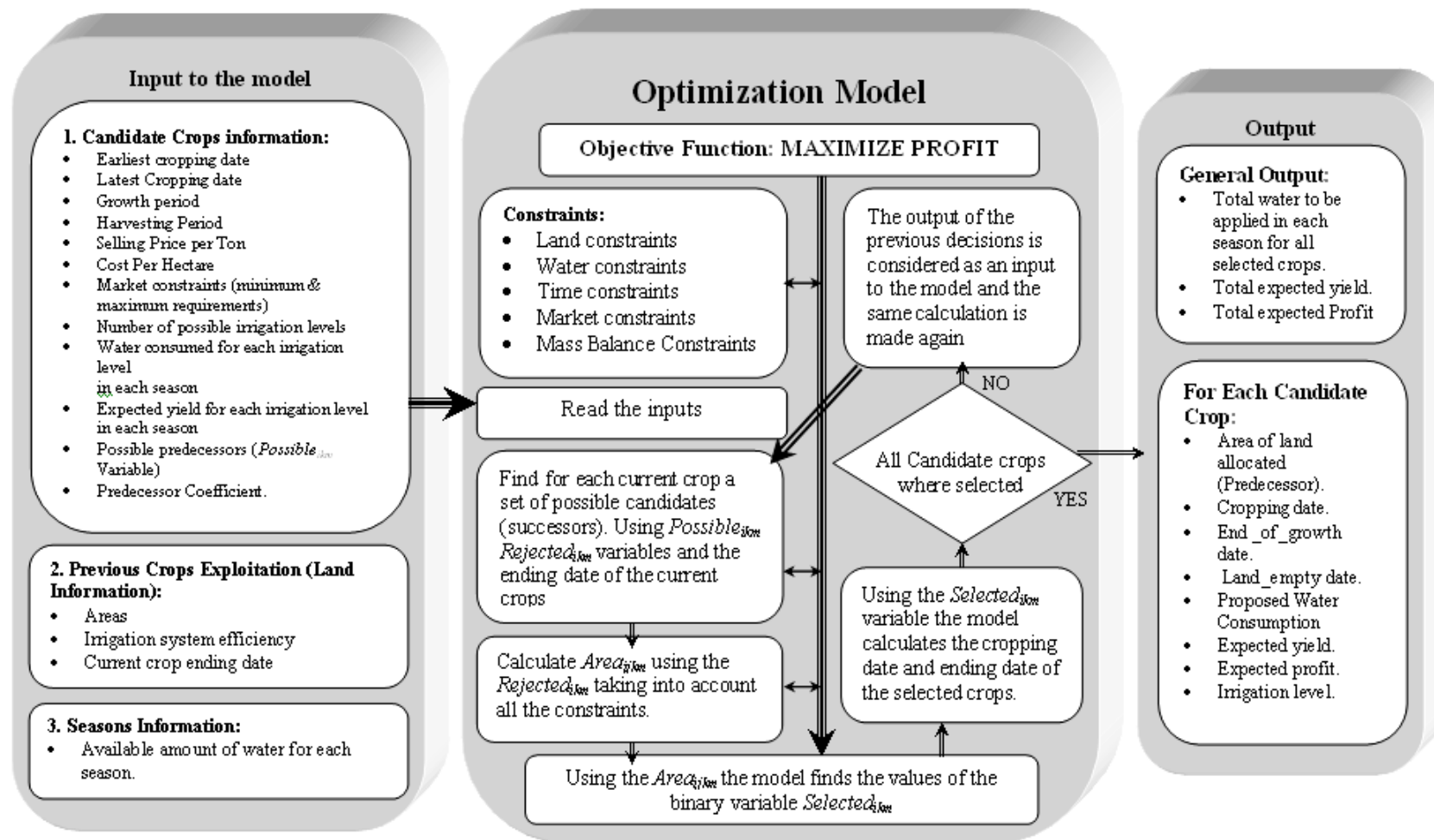


Figure 6.1: optimization model flow chart.

# Chapter 7

---

## Irrigation Schedule Model

### 7.1. Overview of the model

This model translates the output of the single crop model and multi crop model into readable information by the farmer or by normal user, without the need to go in the details through the optimization procedure, irrigation levels, and all other information used to get the output.

### 7.2. The input for this model

- Cropping date,
- Duration (days) for each growth stage,
- Selected irrigation level for each growth stage.
- Daily water consumption per hectare associated with each selected irrigation level.
- Area of land to be cropped

### 7.3. The output of this model

The output of this model will be organized as periods (stages), each has the following information:

- Period No.
- Starting date,
- Ending date,
- Water to be applied per hectare during the whole period
- Total water applied (for the whole cropped area) during the period.

#### 7.4. Notations

$CD_c$ :	Cropping date for crop c. [Input]
$D_t$ :	Duration (days) for stage (t) that is calculated by the yield module. [Input]
$WApPerH_t$ :	Daily water requirement for each hectare during period (t). [Input]
$CA_c$ :	Cropped area by crop ©. [Input]
$SD_t$ :	Starting date of Period (t). [Calculated]
$ED_t$ :	Ending date of period (t). [Calculated]
$Eff$ :	Efficiency of the irrigation system
$LR$ :	Leaching requirements (depends on the water quality)
$TWApPerIrr_t$ :	Total water applied during period (t) for all cropped area. [Calculated]

#### 7.5. Formulation

The calculation procedure is as follows:

For each period, starting from period one, we calculate the starting date. The starting date of the first stage is the cropping date of the crop of interest.

$$SD_1 = CD_c \quad (7.1)$$

The starting date for any stage ( $t > 1$ ) of the crop life is:

$$SD_t = ED_{t-1} + 1 \quad (7.2)$$

The ending date of any stage (t) is calculated as follows:

$$ED_t = SD_t + D_t \quad (7.3)$$

Water requirement per hectare during period (t) is calculated as follows:

$$WApPerH_t = WC_{ij} \times D_t \div [Eff \times (1 - LR)] \quad (7.4)$$

Total water applied during period (t) for all cropped area:

$$TWAp = WApPerH_i \times Area \quad (7.5)$$

The previous procedure can be illustrated by the following example:

**Example 7.1**

The following table represents the cropping date and stage durations for the Green maize growth stages:

Crop Name	Cropping Date (Month/Day/Year)	Durations (days)			
		Stage 1	Stage 2	Stage 3	Stage 4
Grain maize	03/01/2005	20	40	50	30

**Table 7.1: Cropping date and stages duration for Green Maize.**

The selected irrigation levels and the associated expected yield is given in the following table:

Total expected yield (ton)	Expected yield per hectare (ton/Ha)	Land Used (Ha)	Optimal Irrigation levels					
			Stage 1,1	Stage 1,2	Stage 2	Stage 3,1	Stage 3,2	Stage 4
282,157	15,252	18.5	1	1	1	1	0.5	0.5
Daily water Consumption (m3/Ha/day)			20	20	48	85	42	41

**Table 7.2: Land used and selected irrigation levels for Green Maize.**

The output according to the proposed procedure will be:

Period No.	From Date (m/d/y)	To date (m/d/y)	Duration (Days)	Water Required (m <sup>3</sup> /Ha)	Total Water Required (m <sup>3</sup> )	Land Used (Ha)	Expected yield per hectare (ton/Ha)	Total expected yield (ton)
1	03/01/2005	03/14/2005	14	443	8,192			
2	03/15/2005	03/20/2005	6	190	3,511			
3	03/21/2005	04/29/2005	40	3,043	56,292			
4	04/30/2005	05/14/2005	15	2,015	37,274			
5	05/15/2005	06/18/2005	35	2,351	43,486			
6	06/19/2005	07/18/2005	30	1,935	35,797			

**Table 7.3: Output of the irrigation schedule model.**

## Chapter 8

---

# Conclusions and Directions for Future Research

### 8.1. Conclusions

In this study, we developed three models of optimal water allocation with deficit irrigation in order to determine the optimal cropping plan for a variety of scenarios. The first two models deal with a single crop in both cases of limited and unlimited land. The third model deals with the situation where multiple crops are competing for land and water. Applying deficit irrigation efficiently requires taking into account the sensitivity of the different growth stages of a crop to water stress. This has been approached using Dynamic programming. The problem is further complicated by the seasonal supply of water which constrains the ability to perform the best tradeoffs of water allocation that consists essentially on giving the most sensitive growth periods the least deficit and vice versa. The proposed approach used several submodels that interface with each other and with the DP to generate the optimal solution. The output of the single crop model shows in particular that deficit irrigation is beneficial to some crops and is counter-productive for others. Moreover, the results suggest the best tradeoff between expanding the irrigated area for a given crop and the corresponding water allocation.

The multi-crop model is approached by interfacing an optimization model with the single crop model. An important challenge in this problem is to optimize water and land allocation both in space and time. In fact, the problem deals with selecting crops in a feasible sequence accounting for time constraints of crop lives and also on the adequacy of the sequence, expressed through a crop predecessor coefficient that penalizes inappropriate successors of a given crop in the sequence. The optimization in time also accounts for the

seasonal supply of water and the growth stages of the different candidate crops. The optimization in space deals with determining the optimal land to be used per selected crop. Binary variables are used in the optimization model to control time feasibility. Others are used only for the sake of reducing the computational space. The final output of the multi-crop model takes only few seconds using Lingo 8.0 interfaced with a hypothetical database and a computer code using visual basic 6.0 programming language.

The output of the multi-crop model shows the importance of several factors in producing an optimal cropping plan. These include water availability, crop predecessor coefficient, market and land constraints and profit values of competing crops. A comprehensive sensitivity analysis has been performed to investigate these factors.

An important methodological contribution of this study resides in the fact that the different models developed are such that one model serves for developing the next. In particular, the multi-crop model, which is of primary concern for this study is very difficult to approach without the single crop model, which in turn used extensively the DP.

## **8.2. Direction for Future Research**

The output of the current models suggests the optimal irrigation levels to be applied over a given period of time. However, from a practical point of view, it is better providing the user with the appropriate irrigation schedule rather than only proposing the amount of water to be used. This suggests integrating the current models with a mathematical scheduling model.

This study did not account for the risk factor, which is usually important for decision-makers in agricultural activities (considered to hold a risk-averse attitude). Approaches such as E-V (expected value-less variance), utility theory, or equivalently integrating a risk-aversion coefficient in the objective function will take care of this factor, provided that it is suitably adapted in a way that reflects the true preferences of the decision-maker.

Also, precipitations were not considered in the analysis. Accounting for that suggests using a stochastic approach will add further complications to the system. However, for arid regions such as the Kingdom of Saudi Arabia, precipitation is rare and its effect will be low on the current models.

The current study uses costs as inputs. Costing the cropping activity is however a major handicap for all agricultural companies toward identifying their real profits. An

exhaustive cost analysis for the different cost centers of the user would greatly improve the real decisions of the cropping plans and makes the profit assessment more accurate. This would also be a good area for investigation.

## References:

1. **Al-Harkan, I., Al-Azba A., Hariga, M., Azaiez, M.N., Al-Fawzan, M.,** “Decision Support System for Optimal Use of Irrigation Water and Crop Selection”, Second Detailed Progress Report, King Abdulaziz City for Science and Technology, Project No. AR-19-37, 2001.
2. **Azaiez, M. N.,** “A Model for Conjunctive Use of Ground and Surface Water With Opportunity Costs”, *European Journal of Operational Research*, vol. 143 No. 3. pp. 611-624 (2001).
3. **Azaiez, M. N. and Hariga M.,** A Single-Period Model for Conjunctive Use of Ground and Surface Water Under Severe Overdrafts and Water Deficit, *European Journal of Operational Research*, Vol. 133, No. 3, Pages 653-666, (2001).
4. **Azaiez, M.N., Hariga M., Al-Harkan, I.,** “A Chance- Constrained Multi-Period Model for A Multi-Reservoir System Under Water Deficit and Severe Overdrafts of the Aquifer”, *Computers and Operations Research*, in press (2004).
5. **Arumugam, N., and S. Mohan.** Integrated Decision Support System for Tank Irrigation System Operation. *Journal of water resources planning and management*, Vol.123, No. 5, pages 266-273, September- October (1997).
6. **Doorenbos, J., and Kassam, A. H.,** "Yield Response to Water," *Irrigation and Drainage Paper 33*, Food and Agriculture Organization of the United Nations (FAO), Rome, Italy, 193 p, (1979).
7. **William E. Hart, Donald I. Nourm, and Gideon Per.** Optimal Seasonal Irrigation Application Analysis. *Journal of the irrigation and Drainage Division*, vol. 106, No. IR3, Pages 221-233, September (1980).
8. **Haouari, M., and, Azaiez, M. N.,** Optimal cropping patterns under water deficit, *European Journal of Operational Research*, Vol. 130, No. 1, Pages 133-146, (2001).
9. **Hargraves, G. H., and Samani, Z. A.,** Economic Consideration of Deficit Irrigation, *Journal of Irrigation and Drainage Engineering*, Vol. 110, No. 4, Pages 343-358, December, (1984).
10. **Sunantara, D. J., and Ramirez. A. G.,** Optimal Stochastic Multicrop Seasonal and Intraseasonal Irrigation Control. *Journal of Water Resource Planning and Management*, Vol. 123, No. 1, January/February, (1995).

11. **Reddy, J. M., Wilamowski, B., and Cassel-Shamasarker. F.,** Optimal Scheduling of Irrigation for Lateral Canals. *ICID Journal, Vol. 48. No. 3, pages 1-12, August, (1999).*
12. **Stewart, J. I., and Hagan, E. M.,** Functions to Predict Effects of Crop Water Deficits, *Journal of The Irrigation and Drainage Division, ASCE, Vol. 99, No. IR4, Pages 421-439, December, (1973).*
13. **Vedula, S., Mujumdar, P.P.,** Optimal Reservoir Operation For Irrigation of Multiple Crops, *Water Resource Research, Vol. 28, No. 1, Pages 1-9, (1992)*
14. **Vedula, S., Nagesh Kumar, D.,** An Integrated Model For Optimal Reservoir Operation For Irrigation of Multiple Crops, *Water Resource Research Vol. 32, No. 4, Pages 1101-1108, April, (1996).*
15. **Wardlaw, R., and Barnes. J.,** Optimal Allocation of Irrigation Water Supplies in Real Time, *Journal of Irrigation and Drainage Engineering, Vol. 125, No. 6, Pages 345-354, November/December, (1999).*
16. **Yakowitz, S.** Dynamic Programming Applications in Water Resource, *Water Resources Research, Vol. 18, No. 4, Pages 673-694, August, (1982).*
17. **Yeh, W. W-G.,** Reservoir Management and Operation Models: A-state-of-the-art review, *Water Resource Research, Vol. 21, No. 12, Pages 1797-1818, December, (1985).*
18. **Ziari, H.A., McCarl, B.,** A Nonlinear Mixed Integer Program Model For evaluating Runoff Impoundments For Supplemental Irrigation, *Water Resources Research, Vol. 31, No. 6, Pages 1585-1594, June, (1995).*
- 19- **Nimah M., Al-Samarai F., Bashour I., Quintana E., Abu-khait M.,** Water requirements of important crops in Saudi Arabia, *Ministry of agriculture and water, Riyadh, June, (1986).*

# Appendices

## Appendix A

# Water-yield sub-model

Detailed information regarding the Water-yield sub-model will be given in this appendix.

### 1. Notations:

The various inputs (basic and calculated) and related notations are given in the following table.

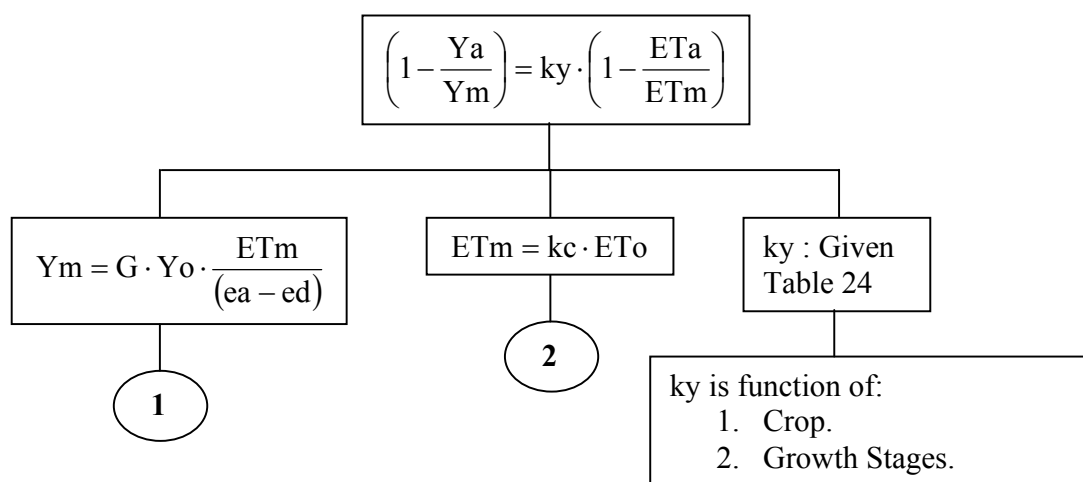
$ET_o$	reference evapotranspiration [ $\text{mm day}^{-1}$ ],	Calculated
$R_n$	net radiation at the crop surface [ $\text{MJ m}^{-2} \text{day}^{-1}$ ],	Input
$G$	soil heat flux density [ $\text{MJ m}^{-2} \text{day}^{-1}$ ],	Input
$T$	mean daily air temperature at 2 m height [ $^{\circ}\text{C}$ ],	Input
$U_2$	wind speed at 2 m height [ $\text{m s}^{-1}$ ],	Calculated
$e_s$	saturation vapour pressure [kPa],	Calculated
$e_a$	actual vapour pressure [kPa],	Calculated
$e_s - e_a$	saturation vapour pressure deficit [kPa],	Calculated
$\Delta$	slope vapour pressure curve [ $\text{kPa } ^{\circ}\text{C}^{-1}$ ],	Calculated
$\gamma$	psychrometric constant [ $\text{kPa } ^{\circ}\text{C}^{-1}$ ],	Calculated
$P$	atmospheric pressure [kPa],	Calculated
$\lambda$	latent heat of vaporization, $2.45 \text{ [MJ kg}^{-1}]$ ,	Input
$c_p$	specific heat at constant pressure, $1.013 \cdot 10^{-3} \text{ [MJ kg}^{-1} \text{ } ^{\circ}\text{C}^{-1}]$ ,	Input
$\varepsilon$	ratio molecular weight of water vapour/dry air = 0.622.	Input
$Z$	elevation above sea level [m],	Input
$E^{\circ}(T)$	saturation vapour pressure at the air temperature $T$ [kPa],	Calculated
$\exp[...]$	2.7183 (base of natural logarithm) raised to the power [...].	Input
$E^{\circ}(T_{\min})$	saturation vapour pressure at daily minimum temperature [kPa],	Calculated
$E^{\circ}(T_{\max})$	saturation vapour pressure at daily maximum temperature [kPa],	Calculated
$RH_{\max}$	maximum relative humidity [%],	Input
$RH_{\min}$	minimum relative humidity [%],	Input
$R_a$	extraterrestrial radiation [ $\text{MJ m}^{-2} \text{day}^{-1}$ ],	Calculated
$G_{sc}$	solar constant = $0.0820 \text{ MJ m}^{-2} \text{min}^{-1}$	Input
$d_r$	inverse relative distance Earth-Sun,	Calculated
$\omega_s$	sunset hour angle [rad],	Calculated
$\varphi$	latitude [rad] (Equation 22),	Calculated
$\delta$	solar declination (Equation 24) [rad].	Calculated
$R_s$	solar or shortwave radiation [ $\text{MJ m}^{-2} \text{day}^{-1}$ ],	Calculated
$R_{so}$	clear-sky solar radiation [ $\text{MJ m}^{-2} \text{day}^{-1}$ ],	Calculated
$R_{ns}$	net solar or shortwave radiation [ $\text{MJ m}^{-2} \text{day}^{-1}$ ],	Calculated
$\alpha$	albedo or canopy reflection coefficient, which is 0.23 for the hypothetical grass reference crop [dimensionless],	Input
$R_{nl}$	net outgoing longwave radiation [ $\text{MJ m}^{-2} \text{day}^{-1}$ ],	Calculated
$\sigma$	Stefan-Boltzmann constant [ $4.903 \cdot 10^{-9} \text{ MJ K}^{-4} \text{m}^{-2} \text{day}^{-1}$ ],	Input
$T_{\max, K}$	maximum absolute temperature during the 24-hour period [ $\text{K} = ^{\circ}\text{C} + 273.16$ ],	Calculated
$T_{\min, K}$	minimum absolute temperature during the 24-hour period [ $\text{K} = ^{\circ}\text{C} + 273.16$ ],	Calculated
$R_s/R_{so}$	relative shortwave radiation (limited to $\leq 1.0$ ),	Calculated

$R_{so}$	calculated (Equation 36 or 37) clear-sky radiation [ $\text{MJ m}^{-2} \text{ day}^{-1}$ ].	Calculated
$c_s$	soil heat capacity [ $\text{MJ m}^{-3} \text{ }^\circ\text{C}^{-1}$ ],	Calculated
$T_i$	air temperature at time $i$ [ $^\circ\text{C}$ ],	Input
$T_{i-1}$	air temperature at time $i-1$ [ $^\circ\text{C}$ ],	Input
$\Delta t$	length of time interval [day],	Input
	$\Delta z$ effective soil depth [m].	Input
$u_z$	measured wind speed at $z$ m above ground surface [ $\text{m s}^{-1}$ ],	Input
	$z$ height of measurement above ground surface [m].	Input
	$a_s + b_s$ fraction of extraterrestrial radiation reaching the earth on clear-sky days ( $n = N$ ).	Input
	$J$ number of the day in the year between 1 and 365 or 366	Input
$Y_a$	actual harvested yield.	Calculated
$Y_m$	maximum harvested yield.	Calculated
$ky$	yield response factor.	Input
$ET_a$	actual evapotranspiration	Calculated
$ET_m$	maximum evapotranspiration.	Calculated
$Y_o$	gross dry matter production of standard crop kg/ha/day.	Calculated
$y_o$	gross dry matter production rate of standard crop for a given location or a completely overcast day kg/ha/day ( <b>Table 3-a3</b> ) and ( <b>Table 3-b3</b> ).	Input
$y_c$	gross dry matter production rates of standard crop for a given location on a clear (cloudless) day, kg/ha/day ( <b>Table 3-a2</b> ) and ( <b>Table 3-b2</b> ).	Input
$F$	fraction of the daytime the sky is clouded, fraction.	Calculated
$S_a$	total available soil water mm/m.	Calculated
$P$	fraction ( $p$ ) of the total available soil water( $S_a$ ).	Input
$D \cdot S_a$	mm/root depth.	Calculated
$kc$	plant coefficient.	Input
$G$	period of growth (days).	Input
$I$	irrigation interval (days).	Calculated
$T$	irrigation interval when $ET_a < ET_m$ (days).	Calculated

**Table AC.1 Inputs and Notations**

## 2. Estimation of the actual yield

The different calculations needed for estimating the actual yield are explained in the following diagram.



**Figure AC.1 Yield calculations**

The details for 1, 2, and 3, of Figure 2.1 are given respectively by Figures 2.2, 2.3, and 2.4.

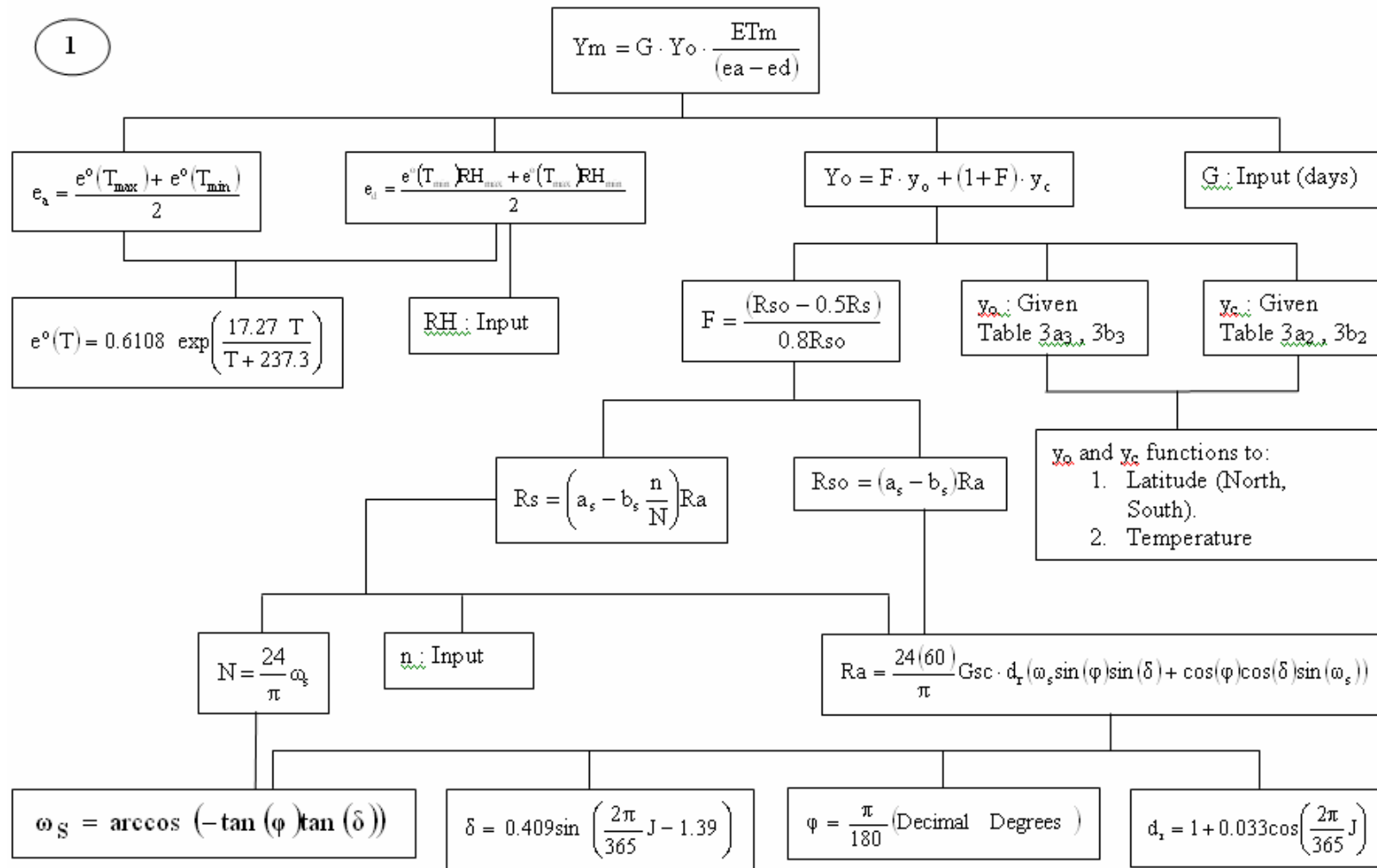
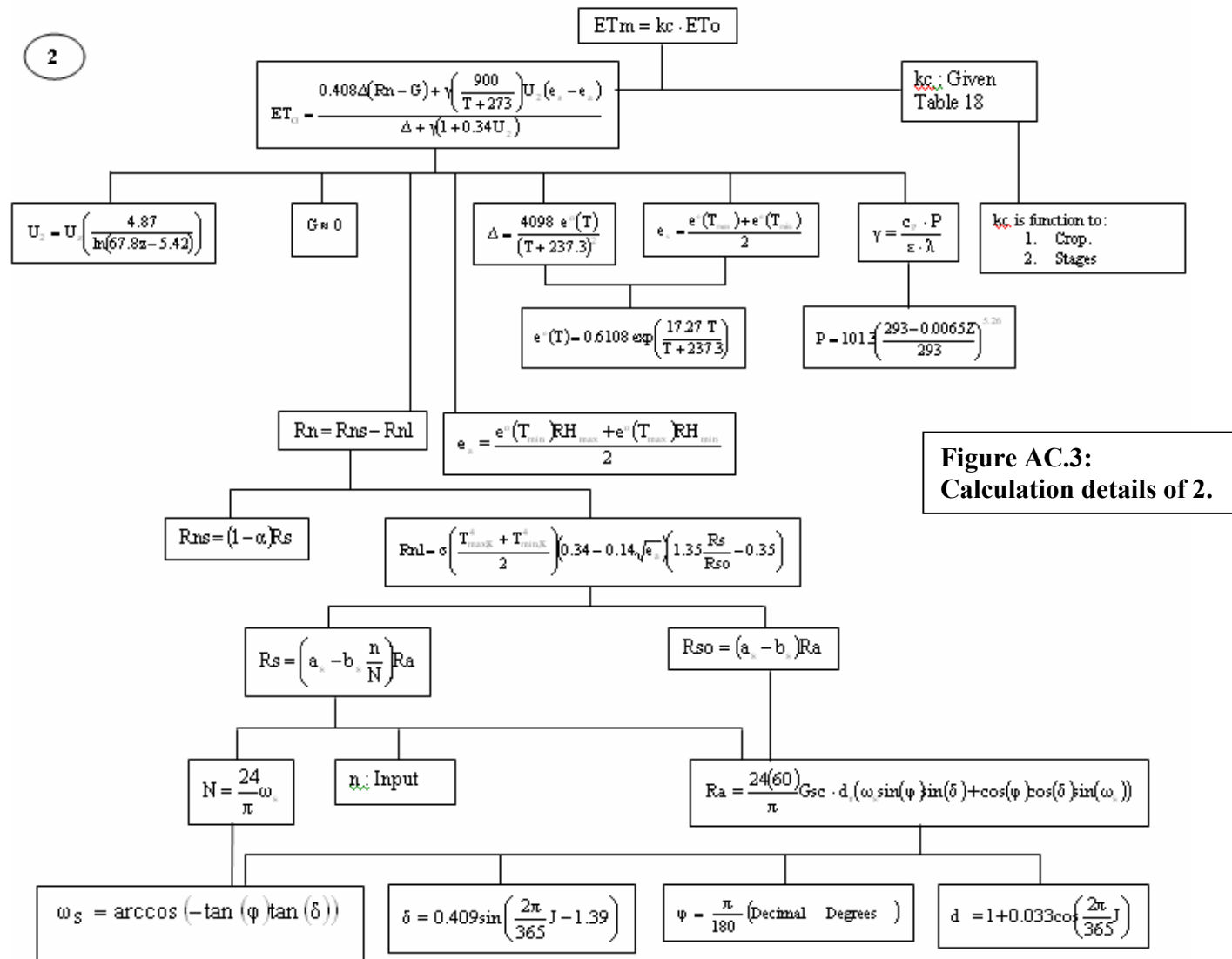


Figure AC.2: Calculation details of 1.



**Figure AC.3:**  
**Calculation details of 2.**

### Example

The following example will consider Green Maize that has the following characteristics:

Green Maize	Stage 1	Stage 2	Stage 3	Stage 4
Duration (days)	20	40	50	30
Starting date	Mar 1	Mar 21	Apr 30	Jun 19
Mid Stage Julian day	70	100	145	185

**Table AA.2: Crop inputs for Green Maize [6]**

The crop is to be cropped in Aljouf region that has the following information:

Region Name	Aljouf
Altitude (Degree)	30.5
Latitude (m)	566
Soil texture	Sandy Loam
Water Quality	Average

**Table AA.3 Region Inputs**

The weather inputs are taken from a weather station in the selected region according to the starting date of the crop and for the duration of each growth stage:

Stage	T <sub>max</sub> (C)	T <sub>min</sub> (C)	T <sub>mean</sub> (C)	Rh <sub>max</sub> (%)	Rh <sub>min</sub> (%)	U <sub>z</sub> (m/Sec)	n (hr)
1	21.57	6.26	13.91	73.45	30.53	1.96	7.71
2	25.52	9.9	17.71	63.79	24.85	1.84	7.43
3	34	17.18	25.63	53.39	20.64	1.5	6.98
4	38.32	20.9	29.61	59.25	22	1.91	8.14

**Table AA.4: Weather station inputs [6]**

The tabulated values are selected according to the crop name, stages duration, and the corresponding weather data:

Stage	yo	yc	cN	Kc	D	p	Ky	Sa	cH	cL	Gs
1	221	416	0.6	0.4	200	0.088	0.4	0.12	0.4	0.5	0
2	235	439	0.6	0.775	450	0.6	1.5				
3	249	466	0.5	1.125	900	0.425	0.5				
4	258	482	0.5	0.875	900	0.425	0.2				

**Table AA.5: Tabulated values inputs [6]**

*Note: The notations used in the input tables are explained in the appendix.*

The output of the yield sub-model according to the above input information is summarized in the following table:

Crop Name	Maximum yield (Kg/Ha)	Net water requirements (m3/Ha)	Leaching Requirements	Irrigation Efficiency	Total water requirements (m3/Ha)
Green Maize	20541	13216	0.1	0.7	20978

**Table AA.6: Output summary for green maize calculated according to the yield sub-model**

## **Appendix B**

# **Intermediate output for the control sub-model**

The following table represent the intermediate output of the control sub-model discussed in chapter 5.

## Appendix C

# Illustration for Multi-crop model

The following chart explains that the decision to be made on the cropping date of the first crop (crop  $k$ ) may affect the feasibility of being followed by the other crop (crop  $i$ ) as the finishing date of  $k$  may exceed the latest start of  $i$ .

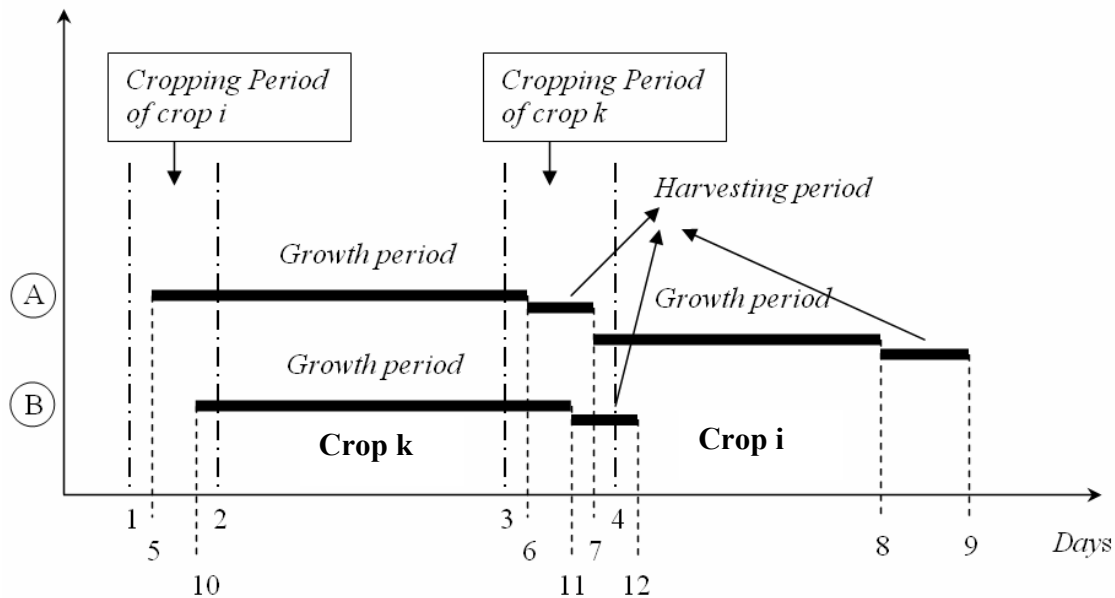


Figure AD.1: Illustrative example for rotation feasibility.

The following will define the numbers mentioned in the above figure (AD.1)

- 1- Earliest cropping date of crop  $i$
- 2- Latest cropping date of crop  $i$
- 3- earliest cropping date of crop  $k$
- 4- Latest cropping date of crop  $k$
- 5- Cropping date of crop  $i$
- 6- End of growth date of crop  $i$

- 7- End of harvest period of crop I (Land is ready for the next crop) and the cropping date of crop k.
- 8- End of growth date of crop k
- 9- End of harvest period of crop k
- 10- Another possible cropping date of crop i
- 11- End of growth date of crop I according to the selected cropping date(10)
- 12- End of harvest date according the selected cropping date (10).

**Case A:** according to the selected cropping date of crop i (point 1), the resulting end of harvest date (point 7) enables to consider the crop k as a feasible successor.

**Case B:** according to the selected cropping date of crop i (point 10), the resulting end of harvest date (point 12) passes the cropping period of crop k. Therefore, although crop k was considered a feasible successor of crop I, but according to the selected cropping date of crop i, crop k became unfeasible successor for crop i.

## Appendix D

# Lingo code for the multi-crop model

The following code is developed implementing the proposed multi-crop model using the extended version of Lingo 8 optimization software.

### The code:

```
Model:
    TITLE KSUPProject4;

Sets:
!Crops:(Primitive set)Contains the crop names of cadidates and current;
!and for the courrent crops the number of irrigation levels is 1 only;
! Crops;
!Candidates:(primitive set) Contains all candidate crops,
including the crrent ones, with thier basic characteristics;
Candidates:C_SeP, C_Ct, C_SDmn, C_SDmx, C_GP, C_MxR,
            C_MnR, C_LC, C_NIL, C_PTR, C_TAC, C_TWC, C_Tey, C_TEP;
!Curren_crops(Candidates)|C_NIL(&1) #EQ# 1;
!Levels:(primitive set) Contains the numbers that
reflects all the possible irrigation levels of the same crop;
Levels_;
!Candidate_Levels: (a Derived, Sparse, Explicit List set)
that is filled from visual basic or execl. contains,
for each crop, all combination of irrigation levels
through all the irrigation Seasons, or in other words
contains all possible candidates of the same crop,
and i will add to it the current crops that has only
one irrigation level;
Candidate_Levels(Candidates, Levels_):
    !CL_EYS1, CL_EYS2, CL_EYS3, CL_EYS4,;
    CL_WCS1, CL_WCS2, CL_WCS3, CL_WCS4,
    CL_EY, CL_WC;
!Crop_Pre:(Derived, dense, explicit list set) contains
the crop predecessor (which are the current crops in
the land under concederation) and also contains the
predecessor coeffecients ;
Crop_Pre(Candidates, Candidates)|C_MxR(&1)#NE#0 :
    CP_PC,Possible ;

!Land:(primitive set)Contains the land information wich
is the current exploitation of crops in addition to all
the candidate crop names that may be crooped on this
land(which are the members of this set) the ending date
of the crop growth and other land related characteristics
as shown below;
```

```

Land(Candidates): L_A, L_CCED, L_IrrSysEff;

tem1_Land_crops(Candidates, Land)|C_MxR(&1)#EQ# 0 #AND#
&1#NE#&2;;

!Land_crops: contains all the possible irrigation choices;
Land_crops(Candidates, Land)|
#NOT# @IN(tem1_Land_crops,&1,&2):LC_A;

tem3_Land_crops(Crop_Pre)|Possible(&1,&2)#EQ#0;;

!Candidates_Land: (Derived dense set) contains all the ;

Candidate_Level_Land(Candidate_Levels, Land_crops)|C_MxR(&1)#GT# 0
#AND# #NOT# @IN(tem3_Land_crops,&1,&3):Area;

Candidate_Land(Candidates, Land_crops)|C_MxR(&1)#GT# 0
#AND# #NOT# @IN(tem3_Land_crops,&1,&2):
CL_ED,CL_CCED,CL_CD,Reject,Selected,production, CL_Area, CL_Profit,
CL_WaterC, CL_IrrEff, CL_FD;

Irr_Seasons/1..4/:S_AW, S_WC;

EndSets

Data:

!the following is to get the result from access instead of excel
that is above;
Candidates, C_SeP, C_Ct, C_SDmn, C_SDmx, C_GP, C_MxR, C_MnR,
C_NIL,C_PTR = @ODBC('KSUProject4','LingoCandidateInformation',
'Crop', 'Expected Selling Price', 'Expected Cost', 'FromD', 'ToD',
'Growth Period', 'Requirement Max', 'Requirement Min',
'Irrigation Levels', 'Harvesting Period');
!Getting the levels;
Levels_ = @ODBC('KSUProject4', 'LingoLevels', 'Levels_');

!Getting the candidates levels information;
Candidate_Levels,
CL_WCS1, CL_WCS2, CL_WCS3, CL_WCS4,
CL_EY, CL_WC = @ODBC('KSUProject4','LingoCandidatesLevels',
'Candidates', 'Levels_',
'CL_WCS1', 'CL_WCS2', 'CL_WCS3', 'CL_WCS4',
'CL_EY', 'CL_WC');

!Getting the Land information;
Land, L_A, L_IrrSysEff, L_CCED = @ODBC('KSUProject4',
'LingoLandInformation', 'Land', 'L_A', 'L_IrrSysEff', 'L_CCED');

!Getting the crop predecessor onformation;
Crop_Pre, CP_PC, Possible = @ODBC('KSUProject4',
'LingoCropPredecessor', 'Candidate', 'Predecessor', 'CP_PC',
'Possible');

!Getting the irrigation season information which is the
amount of water available at each irrigation season;
S_AW = @ODBC('KSUProject4', 'LingoIrrSeasons',

```

```

'Available Water');

!Getting the planning horizon;
PLanning Horizon = @ODBC('KSUPProject4', 'LingoPlanningHorizon',
'Planning Horiazon','Planning Horiazon');

!this is to send the stautus of the sovlver to Visual basic;
@Pointer(1)=@Status();

!this is to transfer the results to access;
@ODBC('KSUPProject4','Candidate_Level_Land',
'Candidates','Levels_','Immediate Predecessor'
,'Land','Area')=
Candidate_Level_Land, Area;

@ODBC('KSUPProject4','Candidate_Land','Candidates',
'Immediate Predecessor',
'Land','Cropping Date','Ending Date','Expected yield',
'Area to be cropped','Water Concumed',
'Irrigation Efficiency','Expected Profit','Finishing date')
= Candidate_Land, CL_CD, CL_ED, production, CL_Area, CL_WaterC,
CL_IrrEff, CL_Profit, CL_FD!, CL_Level; ;
!Candidates: C_TAC, C_TWC, C_Tey, C_TEP;

@ODBC('KSUPProject4', 'Candidates Output','Candidates',
'Total Area Cropped','Total Water Consumed',
'Total Expected Yield','Total Expected Profit')=
Candidates, C_TAC, C_TWC, C_Tey, C_TEP;

!this will send the water used in each season to the DB;
@ODBC('KSUPProject4','LingoIrrSeasons','WaterToBeApplied')=S_WC;

EndData

!Objective function: ;
Max= @Sum(Candidate_Level_Land(i,j,k,m): !Equation (45);
Area(i,j,k,m)*(CP_PC(i,k)*CL_EY(i,j)*C_Sep(i)-C_Ct(i))
);

!1.Market constraints: The total yield of each crop should
be between the maximum and minimum market capacity or demand.and (48);

@For(Candidates(i)|C_MxR(i) #GT# 0: !Equations (47) ;
@Sum(Candidate_Level_Land(i,j,k,m):
Area(i,j,k,m) * CP_PC(i,k) * CL_EY(i,j)) <= C_MxR(i)
);

@For(Candidates(i)|C_MxR(i) #GT# 0: !Equations (48) ;
@Sum(Candidate_Level_Land(i,j,k,m):
Area(i,j,k,m) * CP_PC(i,k) * CL_EY(i,j)) >= C_MnR(i)
);

!2.Land constraints: The sum of areas allocated to all crops

```

that follow a certain predecessor over a certain land is less than or equal to the area of land allocated to this predecessor in previous decision.;

```

@For(Land_crops(k,m): !Equations (49) ;
    @Sum(Candidate_Level_Land(i,j,k,m):
        Area(i,j,k,m)) <= LC_A(k,m)
);

@For(Land_crops(k,m) | C_MxR(k) #GT# 0: !Equations (50) ;
    LC_A(k,m) = @Sum(Candidate_Level_Land(k,j,l,m):
        Area(k,j,l,m)
);

@For(Land(m): !Equations (52) ;
    LC_A(m,m)=L_A(m)
);

```

!3.Water Constraints:Total amount of water to be consumed in each irrigation season should not exceed seasonal water availability. water to be consumed during each irrigation season can be calculated as follows;;

```

S_WC(1)=@Sum(Candidate_Level_Land(i,j,k,m): !Equations (53) ;
    Area(i,j,k,m)*CL_WCS1(i,j)/L_IrrSysEff(m)
);

S_WC(2)=@Sum(Candidate_Level_Land(i,j,k,m): !Equations (53) ;
    Area(i,j,k,m)*CL_WCS2(i,j)/L_IrrSysEff(m)
);

S_WC(3)=@Sum(Candidate_Level_Land(i,j,k,m): !Equations (53) ;
    Area(i,j,k,m)*CL_WCS3(i,j)/L_IrrSysEff(m)
);

S_WC(4)=@Sum(Candidate_Level_Land(i,j,k,m): !Equations (53) ;
    Area(i,j,k,m)*CL_WCS4(i,j)/L_IrrSysEff(m)
);

@For(Irr_Seasons(s): !Equations (54) ;
    S_WC(s) <= S_AW(s)
);

```

!4.Time constraints;;

```

@For(Candidate_Land(i,k,m): !Equations (55) ;
    Reject(i,k,m) >= 1-Possible(i,k)
);

@For(Candidate_Land(o,i,m): !Equations (56) ;
    @For(Candidate_Land(i,k,m):
        CL_ED(i,k,m)+1-C_SDmx(o)-1000*Reject(o,i,m) <= 0
));

```

!Cropping date should be greater than or equal to the first

```

starting date of the crop;
@For(Candidate_Land(i,k,m): !Equations (58) ;
      CL_CD(i,k,m) -C_SDmx(i)*Selected(i,k,m)<=0
    );

!Cropping date should be less than or equal to the second
starting date of the crop;
@For(Candidate_Land(i,k,m): !Equations (59) ;
      CL_CD(i,k,m) -C_SDmn(i)*Selected(i,k,m)>= 0
    );

!Cropping date should be greater than the ending date
of the current crop;
@For(Candidates(o)|C_MxR(o)#GT#0: !Equations (60) ;
      @For(Candidate_Land(i,k,m)|Possible(o,i)#NE#0:
        CL_CD(o,i,m)-CL_ED(i,k,m)-1>= -1000*(1-Selected(o,i,m))
      ));

!for all selected crops the finfishing date of the crop is
less than or equal to the time horizon;
@For(Candidate_Land(i,k,m): !Equations (61) ;
      CL_ED(i,k,m)<=Planing_Horizon
    );

!4.Mass balance constraints;

@For(Candidate_Land(i,k,m): !Equations (62) ;
      @Sum(Candidate_Level_Land(i,j,k,m):
        Area(i,j,k,m))<=1000*(1-Reject(i,k,m))
    );

@For(Candidate_Land(i,k,m): !Equations (63) ;
      @Sum(Candidate_Level_Land(i,j,k,m):
        Area(i,j,k,m))<=1000*Selected(i,k,m)
    );

@For(Candidate_Land(i,k,m): !Equations (64) ;
      @Sum(Candidate_Level_Land(i,j,k,m):
        Area(i,j,k,m))>=0.001*Selected(i,k,m)
    );

!The ending date of any candidate crop is the summation of the starting
date and the harvesting period. And it is calculated as follows;;
@For(Candidate_Land(i,k,m): !Equations (65) ;
      CL_ED(i,k,m)=CL_CD(i,k,m)+
      (C_GP(i)+C_PTR(i))*(Selected(i,k,m))
    );

!In the initial conditions when the allocation for the
crops that has to follow the
current crops directly has to be taken, equation (56)
will take the following form;;

@For(Candidates(i)|C_MxR(i)#GT#0: !Equations (66) ;
      @For(Land(m):

```

```

        CL_CCED(i,m,m)+1-C_SDmx(i)-1000*Reject(i,m,m)<=0
    ));

@For(Candidate_Land(i,m,m): !Equations (67) ;
    CL_CCED(i,m,m)=L_CCED(m)
);

!chick this equation if it is valid or not;
@For(Candidates(i) | C_MxR(i) #GT#0: !Equations (68) ;
    @For(Land(m):
        CL_CD(i,m,m)-CL_CCED(i,m,m)-1>= -1000*(1-Selected(i,m,m))
    ));

@For(Candidate_Land(i,k,m): !Equations (69) ;
    CL_FD(i,k,m)=CL_ED(i,k,m)-C_PTR(i)*Selected(i,k,m)
);

@For(Candidate_Land(i,k,m):@BIN(Reject(i,k,m))); !Equations (70) ;
@For(Candidate_Land(i,k,m):@BIN(Selected(i,k,m))); !Equations (71);

!the following is to transfer the area values to the
Candidate land set;
@For(Candidate_Land(i,k,m): !Equations (72) ;
    CL_Area(i,k,m)=@Sum(Levels_(j) | j#LE#C_NIL(i):Area(i,j,k,m))
);

@For(Candidate_Land(i,k,m): !Equations (73) ;
    production(i,k,m)=@Sum(Levels_(j) | j#LE#C_NIL(i):
        Area(i,j,k,m)*(CP_PC(i,k)*CL_EY(i,j)))
);

!this is to calculate the Expected profit for
each selected crop;
@For(Candidate_Land(i,k,m): !Equations (74) ;
    CL_Profit(i,k,m)=@Sum(Levels_(j) | j#LE#C_NIL(i):
        Area(i,j,k,m)*(CP_PC(i,k)*CL_EY(i,j)*C_SeP(i)-C_Ct(i))
    ));

!this is to calculate the water consumed for each
selected candidate crop;
@For(Candidate_Land(i,k,m): !Equations (75) ;
    CL_WaterC(i,k,m)=@Sum(Levels_(j) | j#LE#C_NIL(i):
        (Area(i,j,k,m)/L_IrrSysEff(m))*CL_WC(i,j)
    ));

!this is to send the agregated result to the
data base;
!total area consumed;
@For(Candidates(i): !Equations (76) ;
    C_TAC=@sum(Candidate_Land(i,k,m):
        CL_Area(i,k,m)
    ));

@For(Candidates(i): !Equations (77) ;
    C_TEY=@sum(Candidate_Land(i,k,m):
        Production(i,k,m)
    ));

```

```

@For(Candidates(i): !Equations (78) ;
      C_TEP=@sum(Candidate_Land(i,k,m):
                  CL_Profit(i,k,m)
                ));

@For(Candidates(i): !Equations (79) ;
      C_TWC=@sum(Candidate_Land(i,k,m):
                  CL_WaterC(i,k,m)
                ));

!this is to transfer the irrigaton efficiency to the
candidate set;
@For(Candidate_Land(i,k,m): !Equations (80) ;
      CL_IrrEff(i,k,m)=L_IrrSysEff(m)*Selected(i,k,m)
    );
End

```

## 1. Sensitivity analysis on Water availability

Land	L_A	End of Growth	Harvesting period (Days)	L_CCED	L_IrrSystem	L_IrrSysEff
Empty	60	01/01/2004	0	1	Pivot system_2	0.95
Alfalfa	60	10/20/2004	15	294	Pivot system_2	0.93
Potato	60	11/15/2004	17	320	Pivot system_2	0.96

**Table S.1: Land Information**

Season	Starting Date	Total Available water (m3)	Water to be consumed (m3)
1	01-Oct	13000	13000
2	01-Dec	50000	49999
3	15-Mar	30000	21903
4	15-May	30000	29999

**Table S.2: Seasonal water consumption of optimal exploitation**

rop Name	Earliest Cropping date	Latest Cropping date	Expected Selling Price (SR/Ton)	Expected Cost (SR/hHa)	Growth Period (Days)	Harvesting Period (Days)	Requirement Max (Ton)	Requirement Min (Ton)	Possible Irrigation Levels
Grain maize	60	74	1500	16000	140	15	200	70	20
Onion	288	305	1700	22000	100	12	150	100	20
Tomato_1	15	46	2000	15000	100	20	100	20	20
Wheat	349	365	1500	500	120	20	400	50	20

**Table S.3: Crop Information**

Candidates	Total Area Cropped (Ha)	Total Water Consumed (m3)	Total Expected Yield (Ton)	Total Expected Profit (SR)
GRAIN_MAIZE	5.28	49916	108	78172
ONION	5.17	13000	123	95849
TOMATO_1	1.39	6012	20	19219
WHEAT	13.97	45973	260	384061

**Table S.4: General output of the optimal exploitation**

Candidates	Immediate Predecessor	Land	Area to be cropped (Ha)	Cropping Date	Harvesting date	Ending Date	Water Consumed (m3)	Irrigation Efficiency	Expected yield (Ton)	Expected Profit (SR)	Irrigation Level
GRAIN_MAIZE	EMPTY	EMPTY	5.28	60	200	215	49916	0.95	108.00	78172.00	20
ONION	EMPTY	EMPTY	5.17	305	405	417	12999	0.95	123.00	95849.00	20
TOMATO_1	EMPTY	EMPTY	1.39	15	115	135	6012	0.95	20.00	19219.00	20
WHEAT	EMPTY	EMPTY	13.97	349	469	489	45973	0.95	260.00	384062.00	12

**Table 6.5: Detailed output of the optimal exploitation**

Candidate	Predecessor	CP_PC	Possible
Grain maize	Grain maize	0.2	1
Grain maize	Wheat	0.8	1
Grain maize	Onion	0.8	1
Grain maize	Tomato_1	0	0
Grain maize	Empty	1	1
Grain maize	Alfalfa	0.7	1
Grain maize	Potato	0.7	1
Wheat	Grain maize	0.9	1
Wheat	Wheat	0.7	1
Wheat	Onion	0.7	1
Wheat	Tomato_1	0.9	1
Wheat	Empty	1	1
Wheat	Alfalfa	0.6	1
Wheat	Potato	0.5	1
Onion	Grain maize	0.8	1
Onion	Wheat	0.9	1
Onion	Onion	0.5	1
Onion	Tomato_1	0.9	1
Onion	Empty	1	1
Onion	Alfalfa	0.8	1
Onion	Potato	0.6	1
Tomato_1	Grain maize	0.8	0
Tomato_1	Wheat	0	0
Tomato_1	Onion	0.9	1
Tomato_1	Tomato_1	0.9	1
Tomato_1	Empty	1	1
Tomato_1	Alfalfa	0.9	1
Tomato_1	Potato	0.8	1

**Table S.6: Crop predecessor coefficient Information**

Season	Starting Date	Total Available water (m3)	Water to be consumed (m3)
1	01-Oct	14000	14000
2	01-Dec	15000	15000
3	15-Mar	30000	21903
4	15-May	30000	29999

**Table S.7: Seasonal water consumption of optimal exploitation**

Candidates	Total Area Cropped (Ha)	Total Water Consumed (m3)	Total Expected Yield (Ton)	Total Expected Profit (SR)
GRAIN_MAIZE	5.28	49916	108	78172
ONION	5.56	14000	132	103222
TOMATO_1	1.39	6012	20	19219
WHEAT	3.33	10973	62	91674

**Table S.8: General output of the optimal exploitation**

Candidates	Immediate Predecessor	Land	Area to be cropped (Ha)	Cropping Date	Harvesting date	Ending Date	Water Consumed (m3)	Irrigation Efficiency	Expected yield (Ton)	Expected Profit (SR)	Irrigation Level
GRAIN_MAIZE	EMPTY	EMPTY	5.28	60	200	215	49916	0.95	108.00	78172.00	20
ONION	EMPTY	EMPTY	5.56	305	405	417	14000	0.95	132.00	103222.00	20
TOMATO_1	EMPTY	EMPTY	1.39	15	115	135	6012	0.95	20.00	19219.00	20
WHEAT	EMPTY	EMPTY	3.33	348	468	488	10973	0.95	62.00	91674.00	12

**Table 6.9: Detailed output of the optimal exploitation**

## 2. Sensitivity analysis on Crop Predecessor coefficient

Candidate	Predecessor	CP_PC	Possible
Grain maize	Grain maize	0.2	1
Grain maize	Wheat	0.8	1
Grain maize	Onion	0.8	1
Grain maize	Tomato_1	0	0
Grain maize	Empty	1	1
Grain maize	Alfalfa	0.7	1
Grain maize	Potato	0.7	1
Wheat	Grain maize	0.9	1
Wheat	Wheat	0.7	1
Wheat	Onion	0.7	1
Wheat	Tomato_1	0.9	1
Wheat	Empty	0.7	1
Wheat	Alfalfa	1	1
Wheat	Potato	0.5	1
Onion	Grain maize	0.8	1
Onion	Wheat	0.9	1
Onion	Onion	0.5	1
Onion	Tomato_1	0.9	1
Onion	Empty	1	1
Onion	Alfalfa	0.8	1
Onion	Potato	0.6	1
Tomato_1	Grain maize	0.8	1
Tomato_1	Wheat	0	0
Tomato_1	Onion	0.9	1
Tomato_1	Tomato_1	0.9	1
Tomato_1	Empty	0.7	1
Tomato_1	Alfalfa	1	1
Tomato_1	Potato	0.8	1

**Table S.10: Crop predecessor coefficient Information**

eason	Starting Date	Total Available water (m3)	Water to be consumed (m3)
1	01-Oct	13000	13000
2	01-Dec	50000	49999
3	15-Mar	30000	21903
4	15-May	30000	29999

**Table S.11: Seasonal water consumption of optimal exploitation**

Candidates	Total Area Cropped (Ha)	Total Water Consumed (m3)	Total Expected Yield (Ton)	Total Expected Profit (SR)
GRAIN_MAIZE	5.28	49916	108	78172
ONION	5.17	12999	123	95849
TOMATO_1	1.98	8589	20	10313
WHEAT	13.36	44912	249	367297

**Table S.12: General output of the optimal exploitation**

Candidates	Immediate Predecessor	Land	Area to be cropped (Ha)	Cropping Date	Harvesting date	Ending Date	Water Consumed (m3)	Irrigation Efficiency	Expected yield (Ton)	Expected Profit (SR)	Irrigation Level
GRAIN_MAIZE	EMPTY	EMPTY	5.28	59	200	215	49916	0.95	108.00	78172.00	20
ONION	EMPTY	EMPTY	5.17	305	405	417	12999	0.95	123.00	95849.00	20
TOMATO_1	EMPTY	EMPTY	1.98	15	115	135	8589	0.95	20.00	10313.00	20
WHEAT	ALFALFA	ALFALFA	13.36	365	485	505	44912	0.93	249.00	367297.00	12

**Table 6.13: Detailed output of the optimal exploitation**

Candidate	Predecessor	CP_PC	Possible
Grain maize	Grain maize	0.2	1
Grain maize	Wheat	0.8	1
Grain maize	Onion	0.8	1
Grain maize	Tomato_1	0	0
Grain maize	Empty	0.9	1
Grain maize	Alfalfa	1	1
Grain maize	Potato	0.7	1
Wheat	Grain maize	0.9	1
Wheat	Wheat	0.7	1
Wheat	Onion	0.7	1
Wheat	Tomato_1	0.9	1
Wheat	Empty	0.7	1
Wheat	Alfalfa	1	1
Wheat	Potato	0.5	1
Onion	Grain maize	0.8	1
Onion	Wheat	0.9	1
Onion	Onion	0.5	1
Onion	Tomato_1	0.9	1
Onion	Empty	1	1
Onion	Alfalfa	0.8	1
Onion	Potato	0.6	1
Tomato_1	Grain maize	0.8	1
Tomato_1	Wheat	0	0
Tomato_1	Onion	0.9	1
Tomato_1	Tomato_1	0.9	1
Tomato_1	Empty	0.7	1
Tomato_1	Alfalfa	1	1
Tomato_1	Potato	0.8	1

**Table S.14: Crop predecessor coefficient Information**

Season	Starting Date	Total Available water (m3)	Water to be consumed (m3)
1	01-Oct	13000	12999
2	01-Dec	50000	50000
3	15-Mar	30000	23418
4	15-May	30000	29999

**Table S.15: Seasonal water consumption of optimal exploitation**

Candidates	Total Area Cropped (Ha)	Total Water Consumed (m3)	Total Expected Yield (Ton)	Total Expected Profit (SR)
GRAIN_MAIZE	5.28	49916	97	61910
ONION	5.17	13000	123	95849
TOMATO_1	1.98	8589	19	10313
WHEAT	13.36	44912	249	367297

**Table S.16: General output of the optimal exploitation**

Candidates	Immediate Predecessor	Land	Area to be cropped (Ha)	Cropping Date	Harvesting date	Ending Date	Water Consumed (m3)	Irrigation Efficiency	Expected yield (Ton)	Expected Profit (SR)	Irrigation Level
GRAIN_MAIZE	EMPTY	EMPTY	5.28	74	214	229	49916	0.95	97.00	61910.00	20
ONION	ALFALFA	ALFALFA	5.06	305	405	417	12999	0.93	120.00	93831.00	20
TOMATO_1	EMPTY	EMPTY	1.98	14	114	134	8589	0.95	20.00	10313.00	20
WHEAT	ALFALFA	ALFALFA	13.36	349	469	489	44912	0.93	249.00	367297.00	12

**Table 6.17: Detailed output of the optimal exploitation**

### 3. Sensitivity analysis minimum market requirement

Season	Starting Date	Total Available water (m3)	Water to be consumed (m3)
1	01-Oct	13000	12999
2	01-Dec	50000	50000
3	15-Mar	30000	18366
4	15-May	30000	29999

**Table S.18: Seasonal water consumption of optimal exploitation**

Crop Name	Earliest Cropping date	Latest Cropping date	Expected Selling Price (SR/Ton)	Expected Cost (SR/hHa)	Growth Period (Days)	Harvesting Period (Days)	Requirement Max (Ton)	Requirement Min (Ton)	Possible Irrigation Levels
Grain maize	60	74	1500	16000	140	15	200	0	20
Onion	288	305	1700	22000	100	12	150	0	20
Tomato_1	15	46	2000	15000	100	20	100	0	20
Wheat	349	365	1500	500	120	20	400	0	20

**Table S.19: Crop Information**

Candidates	Total Area Cropped (Ha)	Total Water Consumed (m3)	Total Expected Yield (Ton)	Total Expected Profit (SR)
GRAIN_MAIZE	5.28	49916	97	61910
ONION	5.06	13000	120	93831
TOMATO_1	0.00	0	0	0
WHEAT	14.41	48450	268	396226

**Table S.20: General output of the optimal exploitation**

Candidates	Immediate Predecessor	Land	Area to be cropped (Ha)	Cropping Date	Harvesting date	Ending Date	Water Consumed (m3)	Irrigation Efficiency	Expected yield (Ton)	Expected Profit (SR)	Irrigation Level
GRAIN_MAIZE	EMPTY	EMPTY	5.28	74	214	229	49916	0.95	97.00	61910.00	20
ONION	ALFALFA	ALFALFA	5.06	305	405	417	12999	0.93	120.00	93831.00	20
WHEAT	ALFALFA	ALFALFA	14.41	365	485	505	48449	0.93	268.00	396226.00	12

**Table 6.21: Detailed output of the optimal exploitation**

Season	Starting Date	Total Available water (m3)	Water to be consumed (m3)
1	01-Oct	13000	13000
2	01-Dec	50000	50000
3	15-Mar	30000	30000
4	15-May	30000	20120

**Table S.22: Seasonal water consumption of optimal exploitation**

Crop Name	Earliest Cropping date	Latest Cropping date	Expected Selling Price (SR/Ton)	Expected Cost (SR/hHa)	Growth Period (Days)	Harvesting Period (Days)	Requirement Max (Ton)	Requirement Min (Ton)	Possible Irrigation Levels
Grain maize	60	74	1500	16000	140	15	200	0	20
Onion	288	305	1700	22000	100	12	150	0	20
Tomato_1	15	46	2000	15000	100	20	100	70	20
Wheat	349	365	1500	500	120	20	400	200	20

**Table S.23: Crop Information**

Candidates	Total Area Cropped (Ha)	Total Water Consumed (m3)	Total Expected Yield (Ton)	Total Expected Profit (SR)
GRAIN_MAIZE	3.54	33478	65	41522
ONION	5.06	13000	120	93831
TOMATO_1	6.93	30062	70	36096
WHEAT	10.88	36579	203	299149

**Table S.24: General output of the optimal exploitation**

Candidates	Immediate Predecessor	Land	Area to be cropped (Ha)	Cropping Date	Harvesting date	Ending Date	Water Consumed (m3)	Irrigation Efficiency	Expected yield (Ton)	Expected Profit (SR)	Irrigation Level
GRAIN_MAIZE	EMPTY	EMPTY	3.54	74	214	229	33478	0.95	65.00	41522.00	20
ONION	ALFALFA	ALFALFA	5.06	305	405	417	12999	0.93	120.00	93831.00	20
TOMATO_1	EMPTY	EMPTY	6.93	15	115	135	30062	0.95	70.00	36096.00	20
WHEAT	ALFALFA	ALFALFA	10.88	365	485	505	36579	0.93	203.00	299149.00	12

**Table 6.25: Detailed output of the optimal exploitation**

#### 4. Sensitivity analysis on Profit

Crop Name	Earliest Cropping date	Latest Cropping date	Expected Selling Price (SR/Ton)	Expected Cost (SR/hHa)	Growth Period (Days)	Harvesting Period (Days)	Requirement Max (Ton)	Requirement Min (Ton)	Possible Irrigation Levels
Grain maize	60	74	1500	16000	140	15	200	0	20
Onion	288	305	1700	22000	100	12	150	0	20
Tomato_1	15	46	2000	15000	100	20	100	0	20
Wheat	349	365	1500	500	120	20	400	0	20

**Table S.26: Crop Information**

Candidates	Total Area Cropped (Ha)	Total Water Consumed (m3)	Total Expected Yield (Ton)	Total Expected Profit (SR)
GRAIN_MAIZE	5.28	49916	97	61910
ONION	5.06	13000	120	93831
TOMATO_1	0.00	0	0	0
WHEAT	14.41	48450	268	396226

**Table S.27: General output of the optimal exploitation**

Season	Starting Date	Total Available water (m3)	Water to be consumed (m3)
1	01-Oct	13000	13000
2	01-Dec	50000	50000
3	15-Mar	30000	29677
4	15-May	30000	29999

**Table S.28: Seasonal water consumption of optimal exploitation**

Crop Name	Earliest Cropping date	Latest Cropping date	Expected Selling Price (SR/Ton)	Expected Cost (SR/hHa)	Growth Period (Days)	Harvesting Period (Days)	Requirement Max (Ton)	Requirement Min (Ton)	Possible Irrigation Levels
Grain maize	60	74	1500	1600	140	15	200	0	20
Onion	288	305	1700	22000	100	12	150	0	20
Tomato_1	15	46	2000	15000	100	20	100	0	20
Wheat	349	365	1500	500	120	20	400	0	20

**Table S.29: Crop Information**

Candidates	Total Area Cropped (Ha)	Total Water Consumed (m3)	Total Expected Yield (Ton)	Total Expected Profit (SR)
GRAIN_MAIZE	10.56	62777	131	180690
ONION	5.06	13000	120	93831
TOMATO_1	0.00	0	0	0
WHEAT	13.95	46900	260	383550

**Table S.30: General output of the optimal exploitation**

Season	Starting Date	Total Available water (m3)	Water to be consumed (m3)
1	01-Oct	13000	12999
2	01-Dec	50000	50000
3	15-Mar	30000	18366
4	15-May	30000	29999

**Table S.31: Seasonal water consumption of optimal exploitation**

Crop Name	Earliest Cropping date	Latest Cropping date	Expected Selling Price (SR/Ton)	Expected Cost (SR/hHa)	Growth Period (Days)	Harvesting Period (Days)	Requirement Max (Ton)	Requirement Min (Ton)	Possible Irrigation Levels
Grain maize	60	74	1500	16000	140	15	200	0	20
Onion	288	305	1700	22000	100	12	150	0	20
Tomato_1	15	46	2000	15000	100	20	100	0	20
Wheat	349	365	1500	500	120	20	400	0	20

**Table S.32: Crop Information**

Candidates	Total Area Cropped (Ha)	Total Water Consumed (m3)	Total Expected Yield (Ton)	Total Expected Profit (SR)
GRAIN_MAIZE	5.28	49916	97	61910
ONION	5.06	12999	120	193949
TOMATO_1	0.00	0	0	0
WHEAT	14.41	48450	268	396226

**Table S.33: General output of the optimal exploitation**

Season	Starting Date	Total Available water (m3)	Water to be consumed (m3)
1	01-Oct	13000	12999
2	01-Dec	50000	9695
3	15-Mar	30000	30000
4	15-May	30000	30000

**Table S.34: Seasonal water consumption of optimal exploitation**

Crop Name	Earliest Cropping date	Latest Cropping date	Expected Selling Price (SR/Ton)	Expected Cost (SR/hHa)	Growth Period (Days)	Harvesting Period (Days)	Requirement Max (Ton)	Requirement Min (Ton)	Possible Irrigation Levels
Grain maize	60	74	1500	16000	140	15	200	0	20
Onion	288	305	1700	22000	100	12	150	0	20
Tomato_1	15	46	2000	15000	100	20	100	0	20
Wheat	349	365	1500	50000	120	20	400	0	20

**Table S.35: Crop Information**

Candidates	Total Area Cropped (Ha)	Total Water Consumed (m3)	Total Expected Yield (Ton)	Total Expected Profit (SR)
GRAIN_MAIZE	5.28	49916	97	61910
ONION	5.06	12999	120	93831
TOMATO_1	4.56	19779	46	23748
WHEAT	0.00	0	0	0

**Table S.36: General output of the optimal exploitation**